Monopolistically Competitive Search Equilibrium*

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Abstract

Labor market intermediaries have been playing an increasingly important role in job matching. We introduce a monopolistically-competitive recruiting (intermediated) sector alongside a standard non-intermediated search and matching model to explore implications of intermediated labor markets. We obtain five main results, three of them analytical and the other two quantitative. First, in the partial equilibrium model, the positive surplus earned by successful monopolistic intermediation appears directly in the surplus-sharing condition between a newly-matched worker and firm. The analytical form of the surplus-sharing condition is unique in that the positive surplus earned by the intermediary appears *additively* — that is, there are *three* distinct surplus components, instead of the standard two distinct components.

The appearance of the additional component is due to a related second analytical result, which is the *aggregate increasing returns in matching* that arise through monopolistic intermediation. The aggregate increasing returns reflect the degree of competition in the intermediated sector. The third analytical result arises in the general equilibrium model, in which the aggregate increasing returns in matching appear as a *resource waste* in the aggregate goods frontier. In terms of quantitative results, the model shows how deviations from efficient wage setting in non-intermediated markets (i.e., departures from the Nash-Hosios condition) and/or changes in the costs of posting vacancies between intermediated and non-intermediated labor markets have important implications for the aggregate behavior of unemployment.

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1 Introduction

Labor market intermediaries (LMIs)—employment agencies and recruiting and staffing firms, online job search engines and services, among others—play an important role in helping firms meet their employment needs and job seekers find employment opportunities.[1] The services and reach of these intermediaries has grown over time, especially with a dramatic expansion of e-recruiting firms and their services since the middle of the 1990s (Nakamura et al., 2009; Bagues and Sylos Labini, 2009).[2] The structure of job matching markets has received little attention in the macro-labor literature, despite the rising prominence of intermediated labor markets and the latter's potentially important consequences for aggregate outcomes.

This paper introduces a monopolistically-competitive recruiting sector with endogenous entry of recruiters into an otherwise standard search and matching model of labor markets. The core of our model builds on the work of Moen (1997) and Shimer (1996), who are the first to characterize *competitive* search equilibrium, and on the endogenous entry model of Bilbiie, Ghironi, and Melitz (2012). The model we develop allows us to answer several questions regarding matching markets and their aggregate implications. More precisely, the issues we address using our framework are: 1) determining the extent to which search behavior and intermediated employment creation behave differently in intermediated labor markets vis-a-vis non-intermediated markets; 2) examining consequences for unemployment; 3) understanding the aggregate macroeconomic implications due to the existence of monopolistic intermediated labor markets; and 4) examining the extent to which barriers to entry in the intermediated markets are influenced by efficiency in other markets.[3]

There are five main results, three of which are analytical and two of which are quantitative. Two of the analytical results arise in the partial equilibrium model and carry through to the baseline general equilibrium framework; in the general equilibrium model, a third analytical result arises. The main two quantitative results from the general equilibrium model pertain to the differential response of intermediated labor markets to deviations from efficient wage determination in non-

[1] For an comprehensive summary of online job services, their proliferation, and their importance, see Nakamura et al. (2009). Among other things, labor market intermediaries build resume databases, provide services that centralize job applications, provide customized matching services for firms, and advertise open employment positions. Well-known online (for-profit) e-recruiting services include Monster.com, Indeedcom, and CareerBuider. Similar services operate in the non-profit realm as well (for example, America's Job Bank).

[2] Monster.com, one of the largest e-recruiting firms in the U.S., started in 1995. For the benefits of e-recruiting (which include the reduction of variable recruiting costs and processing costs, among others), see Nakamura et al. (2009). Survey evidence from the Society for Human Resource Management for 2007 suggests that more than 40 percent of new hires in both the public and private sectors originated from e-recruiting (Nakamura et al., 2009). Using data from iLogos Research, Nakamura et al. (2009) document a sharp expansion in the corporate website employment sections in global 500 companies: while in 1998 these sections represented 29 percent of corporate website use, these sections expanded to 94 percent of website use in 2003. For related work on e-recruiting and the labor market, see Autor, Katz, and Krueger (1998), Kuhn (2003), Kuhn and Skuterud (2004), and Stevenson (2008), among others.

[3] For empirical work on online search and matching efficiency, see Kroft and Pope (2014). For more on the social benefits of e-recruiting and internet-based job intermediation, see Autor (2001).

intermediated markets, and the differential response of unemployment, both static and dynamic, to changes in vacancy posting costs across the two labor markets.

First, the partial equilibrium analysis of the monopolistic recruiting sector uncovers a surplus sharing condition between labor suppliers and labor demanders in which a positive surplus is received by the monopolistic recruiting agency for successful intermediation. While this result may seem unsurprising, the surplus-sharing condition that characterizes our finding is. Indeed, this condition, which arises as a result of imperfect competition in the intermediated labor market, is not one that could be attributed to, say, proportional taxes on labor income, consumption purchases, or on goods-producing firms' profits.[4]. Thus, this result stands in contrast to existing work on the distortionary effects of fiscal policy and labor markets.

To illustrate the economic rationale behind the novel surplus-sharing rule we derive, consider the (qualitative) surplus sharing condition

$$\text{Profit of recruiter} + \text{(1-Share) x Surplus of new employee} = \text{Share x Surplus of new employer}, \tag{1}$$

in which the "Share" term is a scalar between zero and one that measures the percentage of the total surplus from a matched employer-employee pair. The presence of the *additive* term in the surplus sharing condition is novel and unique; we are not aware of such a surplus-sharing condition in the literature. As the qualitative surplus sharing expression (1) shows, the shares of the total surplus received by the employee and the employer sum to one. This in turn raises a natural question regarding the source of the *additional* resources above and beyond those received by the worker and the firm.

These additional resources arise from the *aggregate* increasing returns to scale that naturally emerge in matching in the presence of an endogenous intermediary entry, which is the second main analytical result. Aggregate increasing returns in production arise from the monopolistic nature of the recruiting sector that produces differentiated matches. The idea of aggregate increasing returns when production is differentiated traces back to at least Romer (1987). In general, aggregate increasing returns in matching, as specifically applied to intermediation in labor markets as a result of endogenous intermediary entry, has not appeared in the existing literature.[5] The increasing returns in matching that arise in the partial equilibrium labor market carry through to the general equilibrium model.

The general equilibrium model includes, alongside the intermediated labor market rooted in endogenous recruiting firm entry, a standard (search-based) non-intermediated labor market in

[4]This immediately raises questions regarding the consequences for (labor market) and interventions and optimal fiscal policy

[5]Masters (2007) highlights the fact that matching technologies with increasing returns imply that intermediaries can bring about welfare gains. However, his work does not directly focus on intermediation in labor markets.

which wages are determined via Nash bargaining. Goods-producing firms in the general equilibrium model use capital and labor (obtained via both types of labor markets) to produce output. Within this environment, the presence of positive profits in intermediated labor markets introduces a new element that modifies the allocation of production across different uses — that is, the absorption of output — which is our third analytical result.

Of the two main quantitative results from the general equilibrium model, one regards steady states and the other considers cyclical analysis. First, regarding the long-run, we numerically characterize comparative static steady-state equilibria as the Nash bargaining power of workers in the *non-intermediated* labor market varies. First, search vacancy decisions in the latter are consistent with standard search models: a higher bargaining power leads to increased household search but reduced vacancy creation. In contrast, a non-monotonicity in the search and vacancy behavior in intermediated labor markets arises, where for low levels of workers' bargaining power below the Hosios condition, households focus comparatively more in searching in the non-intermediated markets and reduce search via recruiting firms. In turn, production firms focus comparatively more in vacancy creation via intermediated markets. This behavior changes dramatically as workers' bargaining power surpasses the Hosios condition, so that vacancy creation falls across both markets while households hedge against lower matching probabilities in the non-intermediated market by modifying their relative search behavior across labor markets. This highlights an important implication for the behavior of intermediated labor markets, and ultimately unemployment and participation, as a result of deviations from efficiency in *non-intermediated* markets.

Regarding dynamics, the framework's cyclical results are consistent with broad business cycle patterns in spite of its richer labor market structure: amid TFP shocks, our model generates procyclical labor force participation, consumption, and investment, along with countercyclical unemployment. Importantly, we also show that a steady-state reduction in the flow cost of a vacancy in intermediated labor markets leads to a marginal increase (decrease) in steady state unemployment (participation and output). Conversely, a commensurate reduction in the flow cost of posting a vacancy in non-intermediated labor markets leads to a significant reduction (expansion) in unemployment (output and consumption). This asymmetry in the steady-state consequences of differences in flow vacancy creation costs across labor markets translates into non-negligible implications for cyclical unemployment dynamics. In particular, relative to our baseline economy, an economy with lower flow vacancy posting costs in non-intermediated (intermediated) labor markets exhibits sharper (virtually identical) fluctuations in unemployment amid TFP shocks. Therefore, our findings suggest that, in an environment where intermediated and non-intermediated labor markets coexist, not all vacancies are created equal and differential changes in flow vacancy costs across labor markets can have important implications for unemployment.

Our paper is closest to the literature on "middlemen" in labor markets; some — admittedly non-comprehensive, but nevertheless similar in nature to our interest in intermediation — examples of work in this area are Rubinstein and Wolinksy (1987), Masters (2007), Wright and Wong (2014), and, more recently Farboodi, Jarosch, and Shimer (2017), who highlight the conditions under which intermediaries may arise in equilibrium.[6] Of note, the literature on intermediaries has, for the most part, generally been rooted in partial equilibrium environments. Our work differs from existing work by, among other things, considering a tractable search framework with endogenous participation in which intermediated and non-intermediated labor markets can interact in a general equilibrium setting. Moreover, our results regarding the increasing-returns-to-scale nature of intermediated markets and the effects of deviations from efficiency in non-intermediated markets on intermediated markets, as well as their implications for unemployment and participation, complement existing theoretical work on intermediation in matching markets.

The rest of the paper is organized as follows. Section 2 describes the structure of recruiting markets and the surplus-sharing function that arises as recruiters intermediate frictional labor demand and labor supply. Section 3 then embeds the recruiting sector in a general equilibrium framework. Section 4 contains quantitative results from the general equilibrium model and discusses the intuition behind the results. Section 5 briefly places our main results within the context of existing work on intermediation and matching frictions, and Section 6 concludes. Many of the algebraic derivations are provided in a detailed set of Appendices.

2 Recruiters — Partial Equilibrium

We begin with a partial equilibrium model of the imperfectly competitive recruiting sector with endogenous entry.

2.1 Recruiting Market j

There is a continuum $[0, 1]$ of perfectly-competitive recruiting markets. As shown in Figure 1, in each recruiting market $j \in [0, 1]$, perfectly-competitive recruiting agencies purchase differentiated submarket ij matches and aggregate them using a technological aggregator. As shown in Figure 1, in each recruiting market $j \in [0, 1]$, perfectly-competitive recruiting agencies purchase differentiated submarket ij matches and aggregate them using a technological aggregator. Table 1 shows the several matching aggregators considered in the theoretical and quantitative analysis, and, for reference, Table 2 provides definitions of notation used in the partial equilibrium analysis.

[6] Other related studies on intermediation include Hall and Rust (2002), Hendrickson (2016), who rationalizes the existence of a minimum wage in a model where unions arise as middlemen, Gautier, Hu, and Watanabe (2016), who shows that middlemen can arise in a directed search environment, Chang and Zhang (2016), and Gregor and Menzio

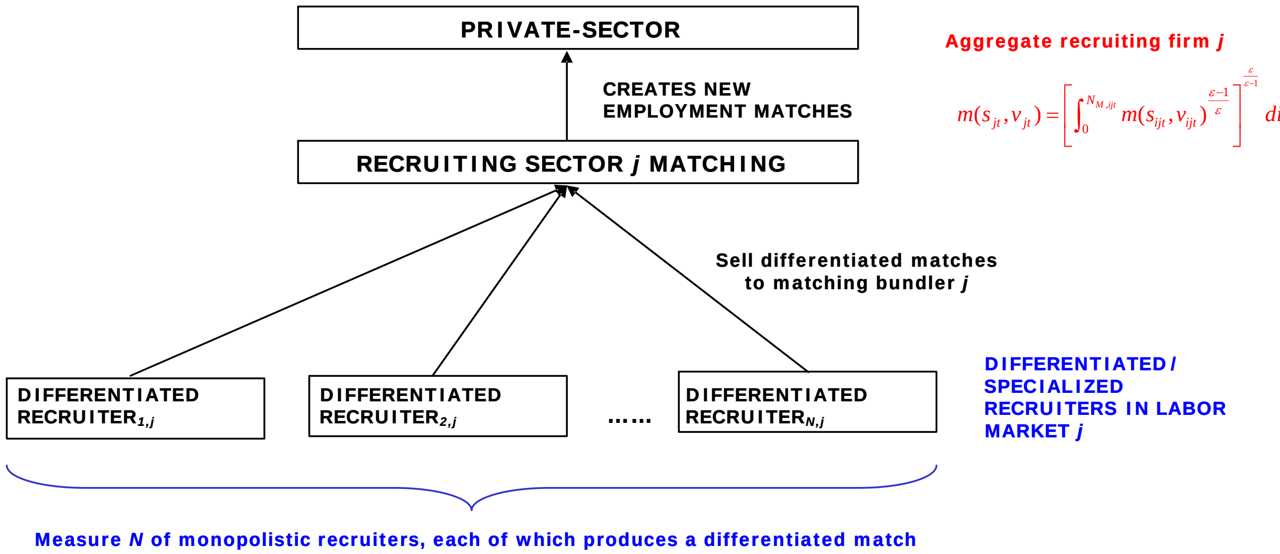

Figure 1: **Structure of Matching Markets.** Differentiated recruiting agencies produce specialized matches in their particular submarkets, which are then aggregated by perfectly-competitive recruiting agencies in labor market j. In each labor market j, there are N_M differentiated recruiting agencies. The matching aggregator displayed is the Dixit-Stiglitz technology.

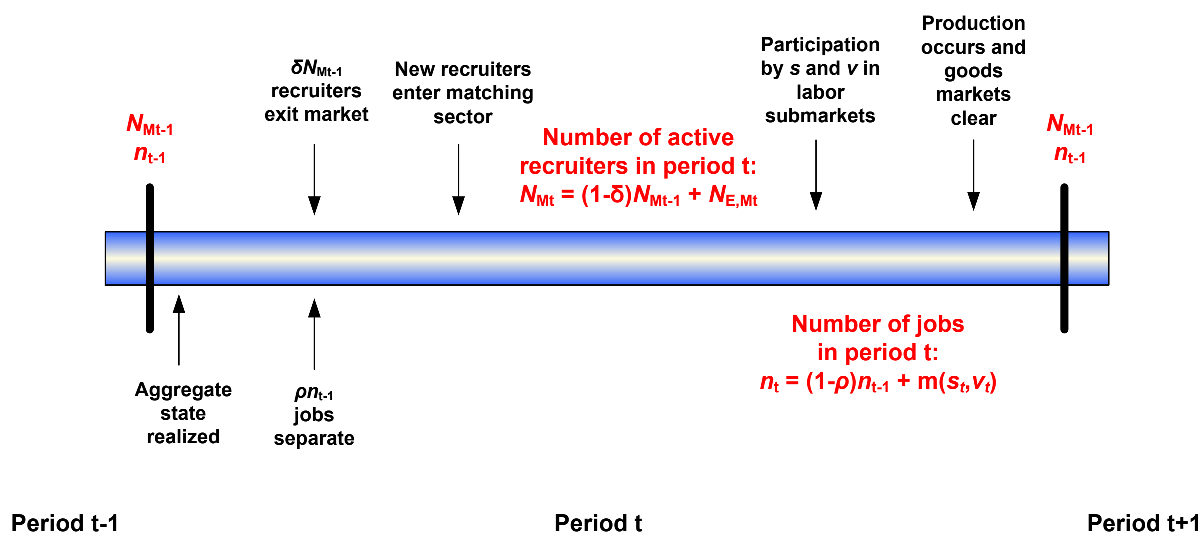

Figure 2: **Ordering of events in intermediated labor markets.** Newly-developed monopolistic recruiting agencies begin operations in period t, and newly-created job matches in period t begin producing goods in period t.

The representative labor-market j recruiting agency is modeled as being a "large" recruiting agency that develops "many" differentiated recruiting agencies. The labor-market j recruiting agency is "large" in the sense that it produces multiple recruiting agencies, but the assumption of a continuum of recruiting firms ensures that each is small relative to the overall labor market, and hence does not internalize the effects of its decisions on the outcomes in matching-market j. Thus, we are assuming that recruiting agency j's decisions regarding the development of new differentiated matching agencies do not internalize the fact that by creating new differentiated matching agencies the profits of *any* existing agencies within the firm are adversely affected (which is dubbed the "profit destruction externality"). This can be rationalized by assuming that new differentiated matching agencies are introduced by independent recruiting line managers who communicate little with each other or are even encouraged to compete with each other.[7]

This rationale allows us to independently characterize the entry of new recruiters in labor market j and the demand for each differentiated recruiter i's match $m(s_{ijt}, v_{ijt})$ in labor market j, to which we now turn.

Entry of New Recruiters.

Expressed in real terms (that is, in units of consumption goods), the intertemporal profit function of the representative recruiter in labor market j is

$$E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left[(\rho_{Mjt} - mc_t)) \cdot N_{Mjt} \cdot m(s_{jt}, v_{jt}) - \Gamma_{Mt} N_{MEjt} \right], \tag{2}$$

in which $\Xi_{t|0}$ is the period-zero discount factor of the ultimate owners of the recruiting firm.[8] The profit function is written in such a way that it anticipates an equilibrium that is symmetric across all submarket match varieties i in labor market j.[9] Entry of a new recruiter in period t entails a sunk cost Γ_{Mt}, which is identical across all potential entrants and is an exogenous process.

The total number of new recruiters in labor market j is N_{MEjt}. The law of motion for the total number of recruiters in labor market j is

$$N_{Mjt} = (1 - \omega) N_{Mjt-1} + N_{MEjt}, \tag{3}$$

(2016), among others.

[7]This assumption is standard in the Bilbiie, Ghironi, and Melitz (2012) class of models on which our recruiting sector builds.

[8]As will be clear in the general equilibrium model in Section 3, the ultimate owner of recruiting firms and hence any flow profits they earn is the representative household.

[9]A priori, the profit function is $\sum_{t=0}^{\infty} \Xi_{t|0} \left[\int_{\Omega_t} (\rho_M(\omega) - mc) \cdot N_{Mt} \cdot m(\rho_M(\omega)) d\omega - \Gamma_{Mt} N_{MEt} \right]$. Supposing there are a measure N_{Mt} of monopolistic recruiters and the matching aggregator is Dixit-Stiglitz, the profit function is $\sum_{t=0}^{\infty} \Xi_{t|0} \left[\int_0^{N_{Mt}} (\rho_M(\omega) - mc_t) \cdot N_{Mt} \cdot m(\rho_M(\omega)) d\omega - \Gamma_{Mt} N_{MEt} \right]$. Then, if the equilibrium creation of new matches is symmetric across all differentiated recruiters, integration leads to the profit function $\sum_{t=0}^{\infty} \Xi_{t|0} \left[(\rho_{Mjt} - mc_t) \cdot N_{Mt} m(s_{jt}, v_{jt}) - \Gamma_{Mt} N_{MEt} \right]$.

Dixit-Stiglitz	Benassy	Translog
$\mu(N_M) = \mu = \frac{\varepsilon}{\varepsilon-1}$	$\mu(N_M) = \mu = \frac{\varepsilon}{\varepsilon-1}$	$\mu(N_M) = 1 + \frac{1}{\sigma N_M}$
$\rho(N_M) = N_M^{\mu-1} = N_M^{\frac{1}{\varepsilon-1}}$	$\rho(N_M) = N_M^{\varphi}$	$\rho(N_M) = \exp\left(-\frac{1}{2}\frac{\tilde{N}_M - N_M}{\sigma \tilde{N} N_M}\right)$
$\epsilon(N_M) = \mu - 1$	$\epsilon(N_M) = \varphi$	$\epsilon(N_M) = \frac{1}{2\sigma N_M} = \frac{1}{2}(\mu(N_M) - 1)$

Table 1: **Matching aggregators.** The markup, relative price of symmetric good, and love of variety as functions of the number of recruiters for the Dixit-Stiglitz, Benassy, and translog variety aggregators. $\tilde{N}_M$ denotes the mass of potential submarket recruiters.

which is a constraint on recruiter j's optimization problem. Given this constraint, recruiter j maximizes its intertemporal profit function (2) by choosing N_{Mjt} and N_{MEjt}. The first-order conditions with respect to N_{Mjt} and N_{MEjt} yield the matching-market j free-entry condition

$$\Gamma_{Mt} = (\rho_{Mjt} - mc_t) \cdot m(s_{jt}, v_{jt}) + (1-\omega)E_t\left\{\Xi_{t+1|t}\Gamma_{Mt+1}\right\}. \tag{4}$$

Intuitively, the free-entry condition equates the marginal cost of entering submarket j to the expected marginal benefit. This expression can be thought of as pinning down the endogenous measure of recruiters N_{Mjt}.

Demand Function for $m(s_{ijt}, v_{ijt})$.

Next, we characterize the representative labor-market j recruiter's demand for $m(s_{ijt}, v_{ijt})$. This characterization requires a reformulation of the profit function stated in (2), the rationale for which is, as described above, the "autonomous" recruiting line managers within the "large" recruiting agency j. The reformulated profit function is the static profit function

$$m(s_{jt}, v_{jt}) - \int_0^{N_{Mijt}} \rho_{M_{ijt}} \cdot m(s_{ijt}, v_{ijt})\, di. \tag{5}$$

For ease of exposition, we assume that the aggregator is of Dixit-Stiglitz form

$$m(s_{jt}, v_{jt}) = \left[\int_0^{N_{Mijt}} m(s_{ijt}, v_{ijt})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di. \tag{6}$$

Taking as given the price ρ_{Mijt} of recruiter ij's output (differentiated match), the representative recruiting agency in labor market j chooses $m(s_{ijt}, v_{ijt})$ to maximize profits

$$m(s_{jt}, v_{jt}) - \int_0^{N_{Mijt}} \rho_{Mijt} \cdot m(s_{ijt}, v_{ijt})\, di, \tag{7}$$

Variable Name	Definitions/Notes
N_{Mjt}	Stock of recruiting agencies in submarket ij
N_{MEjt}	New recruiting agencies in submarket ij
ρ_{Mijt}	Relative price of recruiter ij
w_{ijt}	Wage for newly-hired employees in submarket ij
θ_{ijt}	Labor-market tightness ($\equiv v_{ijt}/s_{ijt}$) in submarket ij
$k^f(\theta_{ijt})$	Probability of job filling in submarket ij
$k^h(\theta_{ijt})$	Probability of job finding in submarket ij
$\mathbf{W}(w_{ijt}, \theta_{ijt})$	Value of active job search participating in submarket ij that successfully finds an employer
$\mathbf{U}_t$	Value of active job search in submarket ij that fails to find a job
$\mathbf{J}(w_{ijt}, \theta_{ijt})$	Value of job vacancy in submarket ij that successfully finds an employee
Γ_{Mt}	Exogenous cost of developing a specialized recruiting agency and entering the recruiting market
ω	Exogenous Poisson exit rate of recruiting agencies

Table 2: **Notation.**

subject to (6). Optimization yields the demand functions

$$m(s_{ijt}, v_{ijt}) = \rho_{Mijt}^{-\varepsilon} \cdot m(s_{jt}, v_{jt}) \tag{8}$$

for each underlying differentiated matching firm ij. Rewriting the demand function to isolate ρ_{Mijt} gives

$$\rho_{Mijt} = m(s_{ijt}, v_{ijt})^{-1/\varepsilon} m(s_{jt}, v_{jt})^{1/\varepsilon}. \tag{9}$$

2.2 Differentiated Recruiter ij

We now turn to the optimization problem of a differentiated recruiter i in labor market j.

Profit Maximization.

As standard in monopolistically competitive models, a differentiated firm (in our application, a differentiated recruiting agency) maximizes profits by choosing its price based on its demand function. Because the matching function $m(s_{ijt}, v_{ijt})$ is constant-returns-to-scale, it is sufficient to describe its cost-per-match in terms of the marginal cost mc_t, which is independent across labor markets. Recruiting agency ij's period-t profits are thus given by

$$\rho_{Mijt} \cdot m(s_{ijt}, v_{ijt}) - mc_t \cdot m(s_{ijt}, v_{ijt}). \tag{10}$$

Continuing with the Dixit-Stiglitz matching aggregator shown in (6), substitution of the Dixit-Stiglitz demand function (8) allows us to rewrite agency ij's period-t profits as

$$(\rho_{Mijt})^{1-\varepsilon} \cdot m(s_{jt}, v_{jt}) - mc_t \cdot (\rho_{Mijt})^{-\varepsilon} \cdot m(s_{jt}, v_{jt}). \tag{11}$$

The first-order condition of (11) with respect to ρ_{Mijt} yields the Dixit-Stiglitz pricing condition

$$\rho_{Mijt} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) mc_t, \tag{12}$$

in which $\mu_t = \frac{\varepsilon}{\varepsilon-1}$ is the constant gross markup that emerges from the Dixit-Stiglitz aggregator. More generally (referring to Table 1), the pricing condition can be expressed as $\rho(N_{Mijt}) = \mu(N_{Mijt}) \cdot mc(N_{Mt})$.

Monopolistic Surplus Sharing.

In terms of the ordering of events (refer to Figure 2), recruiter ij has already maximized profits (and thus minimized costs) before the posting phase (w_{ijt}, θ_{ijt}) that attract both suppliers and demanders to submarket ij. Due to the ordering of events, the posting phase only requires use of recruiter ij's *marginal profit*. More precisely, define the value function associated with the recruiter ij problem as

$$\mathbf{V}_{ij}^{M}(s_{ijt}, v_{ijt}; \cdot) = \rho(N_{Mijt}) \cdot m(s_{ijt}, v_{ijt}) - mc(N_{Mjt}) \cdot m(s_{ijt}, v_{ijt}) - \Gamma_{Mt} \cdot N_{MEijt}, \tag{13}$$

which implies there are two associated envelope conditions. The envelope condition with respect to s_{ijt} is

$$\begin{aligned} \frac{\partial \mathbf{V}_{ij}^{M}(s_{ijt}, v_{ijt}; \cdot)}{\partial s_{ijt}} &= \rho(N_{Mijt}) \cdot m_s(s_{ijt}, v_{ijt}) - mc(N_{Mjt}) \cdot m_s(s_{ijt}, v_{ijt}) \\ &= (\rho(N_{Mijt}) - mc(N_{Mjt})) \cdot \xi \cdot k^h(\theta_{ijt}), \end{aligned} \tag{14}$$

in which the second line follows from the properties of the Cobb-Douglas matching function $m(s, v) = s^{\xi} v^{1-\xi}$. Analogously, the envelope condition with respect to v_{ijt} is

$$\begin{aligned} \frac{\partial \mathbf{V}_{ij}^{M}(s_{ijt}, v_{ijt}; \cdot)}{\partial v_{ijt}} &= \rho(N_{Mijt}) \cdot m_v(s_{ijt}, v_{ijt}) - mc(N_{M,jt}) \cdot m_v(s_{ijt}, v_{ijt}) \\ &= (\rho(N_{Mijt}) - mc_t(N_{Mjt})) \cdot (1-\xi) \cdot k^f(\theta_{ijt}), \end{aligned} \tag{15}$$

in which the second line follows from the properties of the Cobb-Douglas matching function $m(s, v) = s^{\xi} v^{1-\xi}$.

Similar to Moen (1997), recruiter ij has to incentivize both labor suppliers and labor demanders to participate in submarket ij. The incentive mechanism for recruiter ij is to take as constraints participation conditions of labor suppliers and labor demanders. We detail the foundations of the participation constraints in Section 3; for now, we simply describe the participation constraint of a labor supplier as

$$k^h(\theta_{ijt}) \cdot \mathbf{W}(w_{ijt}, \theta_{ijt}) + \Big(1 - k^h(\theta_{ijt})\Big) \mathbf{U} = \mathbf{X}^H \tag{16}$$

and the participation constraint of a labor demander as

$$k^f(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}, \theta_{ijt}) = \mathbf{X}^F. \tag{17}$$

Expression (16) states that the value of a labor supplier directing search towards submarket ij is the same as the value $\mathbf{X}^H$ of directing search to any other submarket. Analogously, expression (17) states that the value of a labor demander that directs its job openings towards submarket ij is the same as the value $\mathbf{X}^H$ of directing its job openings to any other submarket.

Regardless of whether the envelope condition (14) or (15) is used, the following surplus-sharing rule arises.

Proposition 1. ***Monopolistic Surplus Sharing.*** *The surplus-sharing rule between labor suppliers and labor demanders that meet via monopolistically-competitive labor-market intermediation is*

$$\xi \cdot (1-\xi) \cdot (\rho_{Mijt} - mc_t) + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{18}$$

Proof. See Appendix A. □

Observation of the monopolistic surplus sharing rule shows that the percentage of the total surplus received by workers $(1-\xi)$ and the percentage of the total surplus received by goods-producing firms (ξ) sum to 100%. Which then naturally leads to the question of the source of the *extra* resources needed to provide monopolists the *positive economic profit* $\rho_{Mijt} - mc_t$.

2.3 Increasing Returns in Aggregate Matching

The ultimate source is the *increasing returns* that arise in the *aggregate match*. More precisely, substitute the labor market-j matching aggregator (6) into the profit function of the representative labor-market-j recruiter (2).[10] Impose symmetric equilibrium first across all submarkets i in a given labor market j, and then impose symmetric equilibrium across all labor markets j. The aggregate match that arises is

$$N_{Mt} \cdot m(s_t, v_t), \tag{19}$$

in which the N_{Mt} term represents the aggregate increasing returns.

Aggregate increasing returns in production is a well-known idea starting from at least Romer (1987). However, to the best of our knowledge, the aggregate increasing returns in production model has not been applied to recruiting markets in the way in which we do. In our application to recruiting markets, the "production" is production of new job matches using the primitive matching technology. To understand clearly the aggregate increasing returns in matching, examine the recruiting aggregator. Continuing to use the Dixit-Stiglitz aggregator (6) for the sake of simplicity, the *perfectly-competitive* aggregate recruiter j constructs in decentralized labor-market j new job matches via the technology stated in (6). However, if its matching technology were the more general

$$F\left(N_{Mijt}, m(s_{jt}, v_{jt})\right) = \left[\int_0^{N_{Mijt}} m(s_{ijt}, v_{ijt})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di, \tag{20}$$

there are constant returns to scale for the intermediate ij matches in producing the final labor-market match j for a given measure of differentiated recruiters N_{Mijt}. However, there are increasing

[10] Use of the Dixit-Stiglitz aggregator in (6) is sufficient to make the point.

returns to scale once N_{Mijt} is treated as an input argument to production of market-j matches, which implies that operating this $F(.)$ technology in the *perfectly-competitive* labor market j is infeasible.

Monopolistic Wages.

To better describe the implicit monopolistic wage in Proposition 1, we first need to characterize the foundations of the value expressions $\mathbf{W}(w_{ijt})$, $\mathbf{U}_t$, and $\mathbf{J}(w_{ijt})$ and hence the participation constraints. Section 3 provides these foundations, from which wages arising from monopolistic intermediation can be expressed in closed form.

3 General Equilibrium

We now place the partial equilibrium recruiting model into a general equilibrium framework. The general equilibrium framework describes the foundations of the directed search constraints faced by monopolistic recruiters. The general equilibrium framework also allows for non-intermediated matching between labor suppliers and demanders, as is common in macroeconomic models that use the labor search and matching structure. For reference, Table 3 provides definitions of notation for the general equilibrium model.

3.1 Households

There is a continuum $[0, 1]$ of identical households. In each household, there is a continuum $[0, 1]$ of family members. In period t, each family member in the representative household has a labor-market status of employed, unemployed and actively seeking a job, or being outside the labor force. Regardless of which labor-market status a particular family member is in, each family member receives the same exact amount of consumption c_t due to full risk-sharing within each household (see Andolfatto (1996) for formal details).

The representative household maximizes lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\left(n_t + \underbrace{(1 - k_t^{hN}) \cdot s_t^N}_{=ue_t^N} + \int_0^1 \left(\int_0^{N_{M,jt}} \underbrace{(1 - k_{ijt}^h) \cdot s_{ijt}}_{=ue_{ijt}} \, di \right) dj \right) \right] \tag{21}$$

Variable Name	Definitions/Notes
v_{ijt}	Vacancies directed to submarket ij
s_{ijt}	Active job search directed to submarket ij
v_t^N	Vacancies posted in non-intermediated matching market
s_t^N	Active job search in non-intermediated matching market
$\gamma(v_{ijt})$	Vacancy posting cost function for submarket ij
$\gamma_N(v_t^N)$	Vacancy posting cost function for non-intermediated matching market
w_t^N	Wage for employees hired in non-intermediated matching market
θ_t^N	Labor-market tightness $\left(\equiv v_t^N/s_t^N\right)$ in non-intermediated matching market
$k^{fN}(\theta_t^N)$	Probability of v_t^N matching in non-intermediated matching market
$k^{hN}(\theta_t^N)$	Probability of s_t^N matching in non-intermediated matching market
k_t	Physical capital
r_t	Real interest rate
χ	Government-provided unemployment benefits

Table 3: **Notation.**

subject to the budget constraint

$$
\begin{aligned}
c_t + k_{t+1} + T_t = (1 + r_t - \delta)k_t + w_t(1-\rho)n_{t-1} + w_t^N \cdot k_t^{hN} \cdot s_t^N + \int_0^1 \int_0^{N_{M,jt}} w_{ijt} \cdot k_{ijt}^h \cdot s_{ijt} \; di \; dj \\
+ \quad (1 - k_t^{hN}) \cdot s_t^N \chi + \int_0^1 \int_0^{N_{M,jt}} \left(1 - k_{ijt}^h\right) \cdot s_{ijt}\chi \; di \; dj + \int_0^1 \Pi_{jt}^M \; di \cdot dj + \int_0^1 \Pi_{jt}^F \; di \cdot dj
\end{aligned} \tag{22}
$$

and the period-t perceived law of motion of employment

$$
n_t = (1-\rho)n_{t-1} + k_t^{hN} \cdot s_t^N + \int_0^1 \int_0^{N_{M,jt}} k_{ijt}^h \cdot s_{ijt} \; di \; dj. \tag{23}
$$

The optimality conditions (the details of which are provided in Appendix C) that emerge are the standard Euler expression for the supply of physical capital

$$
1 = E_t \left\{ \Xi_{t+1|t} \left(1 + r_t - \delta\right) \right\}, \tag{24}
$$

in which $\Xi_{t+1|t} \equiv \beta u'(c_{t+1})/u'(c_t)$ denotes the stochastic discount factor, and a set of labor-force participation conditions

$$
\frac{h'(lfp_t)}{u'(c_t)} = k_t^{hN} \underbrace{\left[w_t^N + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left(\frac{1 - k_{t+1}^{hN}}{k_{t+1}^{hN}} \right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \right]}_{\equiv \mathbf{W}(w_t^N, \theta_t^N)} + (1 - k_t^{hN}) \underbrace{\chi}_{\equiv \mathbf{U}} \tag{25}
$$

and

$$
\frac{h'(lfp_t)}{u'(c_t)} = k_{ijt}^h \underbrace{\left[w_{ijt} + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left(\frac{1 - k_{jt+1}^h}{k_{jt+1}^h} \right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \right]}_{\equiv \mathbf{W}(w_{ijt}, \theta_{ijt})} + (1 - k_{ijt}^h) \underbrace{\chi}_{\equiv \mathbf{U}} \quad \forall ij. \tag{26}
$$

The participation function (25) characterizes endogenous, but *random*, job search in the non-intermediated labor market, whereas the set of participation functions (26) characterize endogenous *directed* job search towards intermediated labor submarket ij. Given the household-level envelope conditions, around the optimum, active job search in all submarkets must yield the same value $k^h(\theta_{ijt}) \cdot \mathbf{W}(w_{ijt}, \theta_{ijt}) + \left(1 - k^h(\theta_{ijt})\right) \cdot \mathbf{U}(\cdot) = k^h(\theta_{kjt}) \cdot \mathbf{W}(w_{kjt}, \theta_{kjt}) + \left(1 - k^h(\theta_{kjt})\right) \cdot \mathbf{U}(\cdot), \forall i \neq k.$

3.2 Firms

There is a continuum $[0, 1]$ of identical goods-producing firms. The representative goods-producing firm's lifetime profit function is

$$E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left\{ z_t f(k_t, n_t) - r_t k_t - \gamma_N(v_t^N) - \int_0^1 \int_0^{N_{M,jt}} \gamma(v_{ijt}) \; di \; dj \right\}$$

$$- \quad E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left\{ w_t \cdot (1-\rho) n_{t-1} + w_t^N \cdot k_t^{fN} \cdot v_t^N + \int_0^1 \int_0^{N_{M,jt}} w_{ijt} \cdot k_{ijt}^f \cdot v_{ijt} \; di \; dj \right\} \quad (27)$$

subject to the period-t perceived law of motion of employment

$$n_t = (1-\rho) n_{t-1} + k_t^{fN} \cdot v_t^N + \int_0^1 \int_0^{N_{M,jt}} k_{ijt}^f \cdot v_{ijt} \; di \; dj. \quad (28)$$

Profit-maximization (see Appendix B for the formal analysis) leads to the set of job-creation conditions

$$\gamma_N'(v_t^N) = k_t^{fN} \cdot \underbrace{\left(z_t f_n(k_t, n_t) - w_t^N + (1-\rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma_N'(v_{t+1}^N)}{k_{t+1}^{fN}} \right\} \right)}_{\equiv \mathbf{J}(w_t^N, \theta_t^N)}, \quad (29)$$

and

$$\gamma'(v_{ijt}) = k_{ijt}^f \cdot \underbrace{\left(z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma'(v_{jt+1})}{k_{jt+1}^f} \right\} \right)}_{\equiv \mathbf{J}(w_{ijt}, \theta_{ijt})} \quad \forall ij. \quad (30)$$

The job-creation condition (29) characterizes endogenous, but *random*, vacancy postings in the non-intermediated labor market, whereas the set of job-creation condition (30) characterize endogenous *directed* vacancy postings in intermediated labor submarkets ij. Around the optimum, the firm is indifferent between directing new job vacancies to intermediated submarket i or intermediated submarket k, $k^f(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}, \theta_{ijt}) = k^f(\theta_{kjt}) \cdot \mathbf{J}(w_{kjt}, \theta_{kjt})$, $\forall i \neq k$.

3.3 Wage Determination

Wages in Intermediated Labor Market.

With the foundations of the value expressions $\mathbf{W}(w_{ijt})$, $\mathbf{U}_t$, and $\mathbf{J}(w_{ijt})$ for wage determination now in place, we can now state the wage implicit in the monopolistically-competitive surplus sharing condition (18) in explicit form. Substitution of the value expressions $\mathbf{W}(w_t^N)$, $\mathbf{U}_t$, and $\mathbf{J}(w_t^N)$

into (18) yields the (symmetric equilibrium) explicit-form wage

$$w_t = \xi z_t f_n(k_t, n_t) + (1-\xi)\chi + \xi(1-\rho)E_t\left\{\Xi_{t+1|t}\gamma'(v_{t+1})\cdot\theta_{t+1}\right\} \qquad (31)$$
$$- \quad \xi(1-\xi)\left[(\rho(N_{Mt}) - mc(N_{Mt})) - (1-\rho)E_t\left\{\Xi_{t+1|t}\left(\rho(N_{Mt+1}) - mc(N_{Mt+1})\right)\right\}\right],$$

as shown in Appendix D. If the recruiting market is perfectly competitive ala Moen (1997), $\rho_{Mijt} = mc_t\ \forall ijt$, the real wage is characterized by the completely-standard first line (see Arseneau and Chugh, 2012). However, if the recruiting market is imperfectly competitive (and hence $\rho_{Mijt} > mc_t$), then it is not only the period-t profits accruing to the recruiting sector that distort the period-t wage, *period-$(t+1)$ profits also distort the period-t wage*, despite the fact that recruiters only make static decisions. The reason that recruiters' period-$t+1$ rents affect the period-t wage is the long-lasting nature of job relationships, even though monopolistic recruiters themselves do not need employees. Positive recruiter profits, *ceteris paribus*, lead to lower real wages in a *dynamic* sense. To the best of our knowledge, this is a unique and novel form of intertemporal distortion in a dynamic search and matching model.

Nash-Bargained Wages in Non-Intermediated Labor Market.

We assume that the wage model in the non-intermediated labor market is generalized Nash bargaining. Without going into details (which can easily be found in a textbook such as Pissarides (2000, Chapter 1)), the Nash surplus-sharing condition is

$$\mathbf{W}(w_t^N, \theta_t^N) - \mathbf{U}_t = \left(\frac{\eta}{1-\eta}\right)\mathbf{J}(w_t^N, \theta_t^N), \qquad (32)$$

in which $\eta \in (0,1)$ denotes the potential new employee's generalized Nash bargaining power. Substitution of the value expressions $\mathbf{W}(w_t^N, \theta_t^N)$, $\mathbf{U}_t$, and $\mathbf{J}(w_t^N, \theta_t^N)$ yields the explicit-form wage

$$w_t^N = \eta \cdot z_t f_n(k_t, n_t) + (1-\eta)\cdot\chi + \eta(1-\rho)E_t\left\{\Xi_{t+1}\frac{\gamma_N'(v_{t+1}^N)}{k_{t+1}^{fN}}\right\} \qquad (33)$$

in non-intermediated *random search* labor markets.

3.4 Aggregate Employment

The aggregate law of motion for employment

$$n_t = (1-\rho)n_{t-1} + N_{Mt}\cdot m(s_t, v_t) + m(s_t^N, v_t^N) \qquad (34)$$

takes into account both new job matches produced by the intermediated labor market — which, as described in the partial equilibrium model in Section 2, leads to aggregate increasing returns in matching — and the non-intermediated labor market.

3.5 Government

The (symmetric equilibrium) flow budget constraint of the government is

$$T_t = g_t + (1 - k^h(\theta_t)) \cdot s_t \cdot N_{Mt} \cdot \chi + (1 - k^{hN}(\theta_t^N)) \cdot s_t^N \cdot \chi, \tag{35}$$

in which lump-sum taxes T_t levied on households finance government-provided unemployment benefits and government spending g_t.

3.6 Aggregate Goods Resource Constraint

The decentralized economy's aggregate goods resource constraint

$$\begin{aligned} & c_t + k_{t+1} - (1-\delta)k_t + \gamma(v_t) \cdot N_{Mt} \\ + \quad & \gamma_N(v_t^N) + \Gamma_{Mt} N_{MEt} - (\rho_{Mt} - mc_t) \cdot N_{Mt} \cdot m(s_t, v_t) = z_t f(k_t, n_t). \end{aligned} \tag{36}$$

The derivation of the aggregate goods resource constraint (36) appears in Appendix E; here, we simply describe several novel features of the aggregate goods resource constraint (36).

1. The absorption of resources devoted to the recruiting industry, $(\rho_{Mt} - mc_t) \cdot N_{Mt} \cdot m(s_t, v_t)$.

2. The resources devoted to the recruiting industry diminish as $(\rho_{Mt} - mc_t) \to 0$, in which case the *perfectly*-competitive search equilibrium described by Moen (1997) emerges.

3. The appearance of the increasing returns to scale that emerges from the differentiated recruiting sector, captured in the term $N_{Mt} \cdot m(s_t, v_t)$.

3.7 Private-Sector Equilibrium

A symmetric private-sector general equilibrium is made up of sixteen endogenous state-contingent processes $\left\{c_t, n_t, lfp_t, k_{t+1}, N_{Mt}, N_{MEt}, s_t, v_t, \theta_t, w_t, s_t^N, v_t^N, \theta_t^N, w_t^N, \rho_{Mt}, mc_t\right\}_{t=0}^{\infty}$ that satisfy the following sixteen sequences of conditions: the aggregate resource constraint

$$\begin{aligned} & c_t + k_{t+1} - (1-\delta)k_t + \gamma(v_t) \cdot N_{Mt} \\ + \quad & \gamma_N(v_t^N) + \Gamma_{Mt} N_{MEt} - (\rho_{Mt} - mc_t) \cdot N_{Mt} \cdot m(s_t, v_t) = z_t f(k_t, n_t), \end{aligned} \tag{37}$$

the aggregate law of motion for labor

$$n_t = (1-\rho)n_{t-1} + m(s_t^N, v_t^N) + N_{Mt} \cdot m(s_t, v_t), \tag{38}$$

the definition of aggregate LFP

$$lfp_t = N_t + (1-k_t^{hN})s_t^N + (1-k_t^h)s_t, \tag{39}$$

the aggregate law of motion for recruiters

$$N_{Mt} = (1-\omega)N_{M,t-1} + N_{MEt}, \tag{40}$$

the capital Euler condition

$$1 = E_t\left\{\Xi_{t+1|t}\left(1 + z_{t+1}f_k(k_{t+1}, n_{t+1}) - \delta\right)\right\}, \tag{41}$$

the free-entry condition for recruiters

$$\Gamma_{Mt} = (\rho(N_{Mjt}) - mc(N_{Mjt})) \cdot m(s_{jt}, v_{jt}) + (1-\omega)E_t\left\{\Xi_{t+1|t}\Gamma_{Mt+1}\right\}, \tag{42}$$

the vacancy creation condition for intermediated labor markets

$$\gamma'(v_{ijt}) = k^f(\theta_t) \cdot \underbrace{\left(z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\gamma'(v_{jt+1})}{k_{jt+1}^f}\right\}\right)}_{\equiv \mathbf{J}(w_{ijt}, \theta_{ijt})} \;\forall ij, \tag{43}$$

the vacancy creation condition for non-intermediated labor markets

$$\gamma_N'(v_t^N) = k^{fN}(\theta_t^N) \cdot \underbrace{\left(z_t f_n(k_t, n_t) - w_t^N + (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\gamma_N'(v_{t+1}^N)}{k_{t+1}^{fN}}\right\}\right)}_{\equiv \mathbf{J}(w_t^N, \theta_t^N)}, \tag{44}$$

the active job search condition for non-intermediated labor markets

$$\frac{h'(lfp_t)}{u'(c_t)} = k_t^{hN}\underbrace{\left[w_t^N + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(\frac{1-k_{t+1}^{hN}}{k_{t+1}^{hN}}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\}\right]}_{\equiv \mathbf{W}(w_t^N, \theta_t^N)} + (1-k_t^{hN})\underbrace{\chi}_{\equiv \mathbf{U}}, \tag{45}$$

the active job search condition directed towards intermediated labor markets

$$\frac{h'(lfp_t)}{u'(c_t)} = k^h_{ijt} \underbrace{\left[w_{ijt} + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(\frac{1-k^h_{jt+1}}{k^h_{jt+1}}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\}\right]}_{\equiv \mathbf{W}(w_{ijt},\theta_{ijt})} + (1-k^h_{ijt})\underbrace{\chi}_{\equiv \mathbf{U}} \quad \forall ij, \tag{46}$$

the surplus-sharing rule that determines wages w_t in monopolistic labor markets

$$\xi \cdot (\rho_{Mt} - mc_t) + \mathbf{W}(w_t) - \mathbf{U}_t = \left(\frac{\xi}{1-\xi}\right)\mathbf{J}(w_t), \tag{47}$$

the surplus-sharing rule that determines Nash-bargained wages (with η denoting the employee's Nash bargaining power) in non-intermediated labor markets

$$\mathbf{W}(w^N_t) - \mathbf{U}_t = \left(\frac{\eta}{1-\eta}\right)\mathbf{J}(w^N_t), \tag{48}$$

the monopolistic matching-market pricing expression

$$\rho(N_{Mt}) = \mu(N_{Mt}) \cdot mc(N_{Mt}), \tag{49}$$

the definition of labor-market tightness in monopolistic recruiting markets

$$\theta_t = \frac{v_t}{s_t}, \tag{50}$$

the definition of labor-market tightness in monopolistic recruiting markets

$$\theta^N_t = \frac{v^N_t}{s^N_t}, \tag{51}$$

and the symmetric equilibrium ρ_{Mt}

$$\rho_{Mt} = \rho(N_{Mt}), \tag{52}$$

which depends on the particular matching aggregator.

4 Results

4.1 Empirical Targets and Calibration

We assume log utility with respect to consumption, $u(c) = \log c$. In turn, the disutility from participation is given by $h\left(lfp\right) = \kappa(lfp)^{1+\frac{1}{\iota}} / \left(1 + \frac{1}{\iota}\right)$, where $\kappa, \iota > 0$. The production function is Cobb-Douglas, $f\left(k, n\right) = k^{\alpha} n^{1-\alpha}$, with $0 < \alpha < 1$. All matching functions are also Cobb Douglas, $m(s, v) = M \cdot s^{\xi} v^{1-\xi}$ and $m(s^N, v^N) = M_N \cdot \left(s^N\right)^{\xi} \left(v^N\right)^{1-\xi}$, in which ξ is the matching elasticity with respect to active jobs searchers and M and M_N denote, respectively, the exogenous matching efficiency parameters in the intermediated and non-intermediated labor market. This implies that the matching probabilities in the non-intermediated labor market are given by $k^{hN} = m(s^N, v^N)/s^N$ and $k^{fN} = m(s^N, v^N)/v^N$. The corresponding matching probabilities in the intermediated labor market take into account the increasing-returns-to-scale nature of the market, so that $k^h = NM \cdot m(s, v)/s$ and $k^f = NM \cdot m(s, v)/v$. Finally, we allow for the possibility of convex vacancy postings, where the functions $\gamma(\cdot) = \gamma \cdot (v)^{\eta_v}$ and $\gamma_N(\cdot) = \gamma_N \cdot \left(v^N\right)^{\eta_v}$, where $\gamma, \gamma_N > 0$ and $\eta_v \geq 1$.

A period in the model represents a quarter. Following the search and matching and business cycle literatures, we set the capital share to $\alpha = 0.40$, the subjective discount factor to $\beta = 0.99$, the capital depreciation rate $\delta = 0.02$, and the participation elasticity parameter $\iota = 0.18$ (Arseneau and Chugh, 2012). Turning to the labor market parameters, we set the quarterly exogenous separation probability to $\rho = 0.10$, the matching elasticity $\xi = 0.40$, and the Nash bargaining power for workers in non-intermediated labor markets to $\eta = 0.40$. We normalize steady-state aggregate productivity z to 1. Finally, we assume linear vacancy creation costs so that $\eta_v = 1$.

The novel block in our model pertains to the inclusion of imperfectly-competitive intermediation in one of the two matching markets. As a starting point, we set the elasticity of substitution $\varepsilon = 6$ and the the exit rate of recruiting firms $\omega = 0.05$.

We initially assume that $\gamma = \gamma_N$. Then, we calibrate the remaining parameters $\gamma(= \gamma_N)$, χ, κ, M, M_N, and Γ_{Mt} to match the following steady-state targets: a job-finding probability in the non-intermediated market of 0.6, a job-filling probability in the non-intermediated market of 0.7, a labor force of 0.74, a value for unemployment benefits representing 0.40 of average wages, a share of intermediated-market matches in total matches of 0.40, and an entry cost of 0.1. This calibration implies that the total resource cost from vacancy postings and recruiting-firm creation is close to 5 percent of total output (see, for example, Arseneau and Chugh (2012) for a model without recruiting firms). For ease of reference, Table 4 summarizes the baseline parameters. A point that will be discussed further in Section 5 is that the baseline parameters are *not* chosen in a way that purposefully allows for endogeneity of the intermediated labor market. Endogeneity of the monopolistic recruiting sector is an inherent property of the general equilibrium model.

Parameter	Value	Description
		Recruiting Sector
ε	6	Elasticity of substitution for Dixit-Stiglitz aggregator
σ	7.1	Calibrating parameter for translog aggregator
ω	0.05	Quarterly exogenous exit rate of recruiters
		Utility
β	0.99	Quarterly subjective discount factor
κ	4.58	Scaling parameter for $h(.)$
ι	0.18	Wage elasticity of lfp
		Goods Production
α	0.40	Elasticity of Cobb-Douglas goods production function $f(k,n)$ with respect to k
δ	0.02	Quarterly depreciation rate of physical capital
		Labor Market
ρ	0.10	Quarterly exogenous separation of jobs
ξ	0.40	Elasticity of Cobb-Douglas matching technology $m(s,v)$ with respect s
η	0.40	Generalized Nash bargaining power for workers in non-intermediated labor markets

Table 4: **Baseline Parameters.**

4.2 Steady-State Analysis

To understand how potential asymmetries between intermediated and non-intermediated labor markets affect labor market and macro outcomes, consider a change in the Nash bargaining power of workers in the non-intermediated labor market. As shown in Figures 3, 4, and 5, larger bargaining power for workers generates monotonic increases in unemployment and search in non-intermediated labor markets as well as monotonic reductions in vacancy postings and market tightness. These results are well known from standard search models and are intuitive: higher bargaining power implies that households extract a larger share of the surplus from employment relationships, which leads to not only increased household search behavior but also to a reduction in firms' incentive to create vacancies and ultimately sectoral market tightness.

In contrast, the intermediated labor market exhibits non-monotonic changes in its corresponding variables (this also applies to labor force participation). More importantly, the inflection point occurs *when the Hosios condition in the non-intermediated market holds* ($\xi = \eta = 0.4$). In particular, for low levels of the bargaining power, intermediated-market tightness and vacancies (searchers) are increasing (increasing, under Dixit-Stiglitz) in the bargaining power of workers. Conversely, for high levels of the bargaining power, intermediated-market tightness and vacancies (searchers) are decreasing (increasing, under Dixit-Stiglitz) in the bargaining power. In addition, the number of incumbent and new recruiting firms is decreasing in the workers bargaining power.

The inverse-U shaped behavior of labor force participation can be explained by the consistent rise in searchers in non-intermediated markets coupled with the contraction in vacancies (and ultimately employment) as the bargaining power of workers approaches $\eta = 1$. For low levels of worker bargaining power, increased search puts upward pressure on participation while employment is not as responsive, whereas for high levels for the bargaining power, the adverse effect of lower vacancies becomes stronger, leading to an eventual reduction in participation despite the rise in searchers.

The rationale behind the non-monotonic pattern in intermediated-market searchers and vacancies is as follows: as the bargaining power in the non-intermediated market increases from an initially low level, households prefer to direct their search towards the market where their share of the surplus is expanding (i.e., the non-intermediated market). As a result, search in intermediated markets falls.

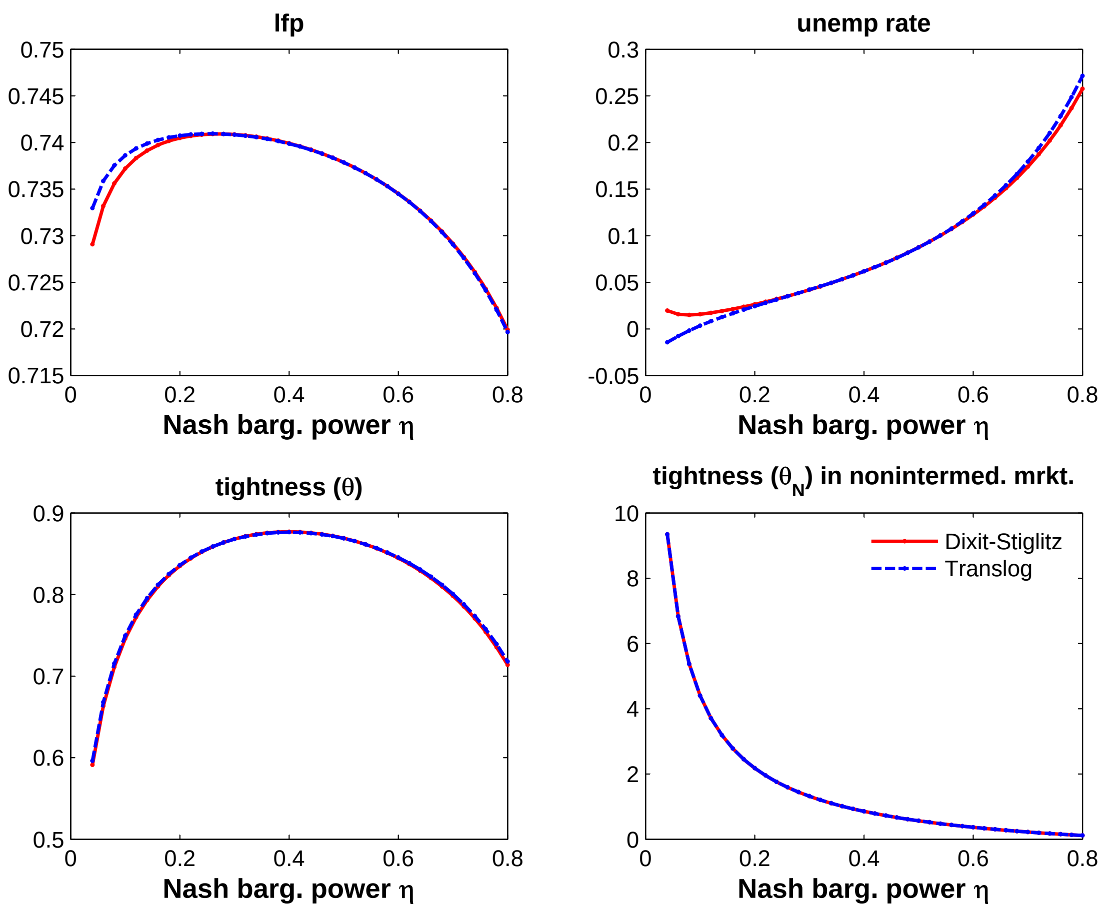

Figure 3: **Steady state as function of worker Nash wage bargaining power η in non-intermediated labor market I.** Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation. All other parameters besides η are held at their baseline values.

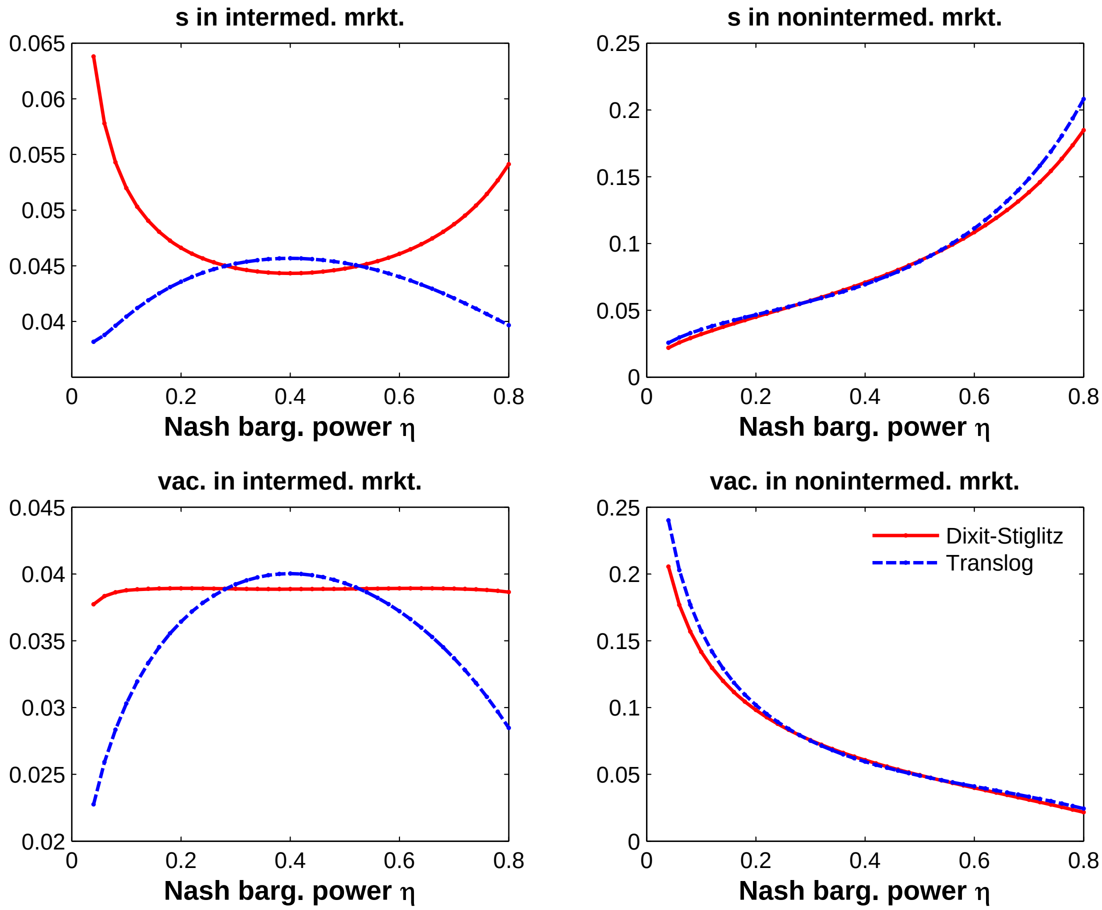

Figure 4: **Steady state as function of worker Nash wage bargaining power η in non-intermediated labor market II.** Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation. All other parameters besides η are held at their baseline values.

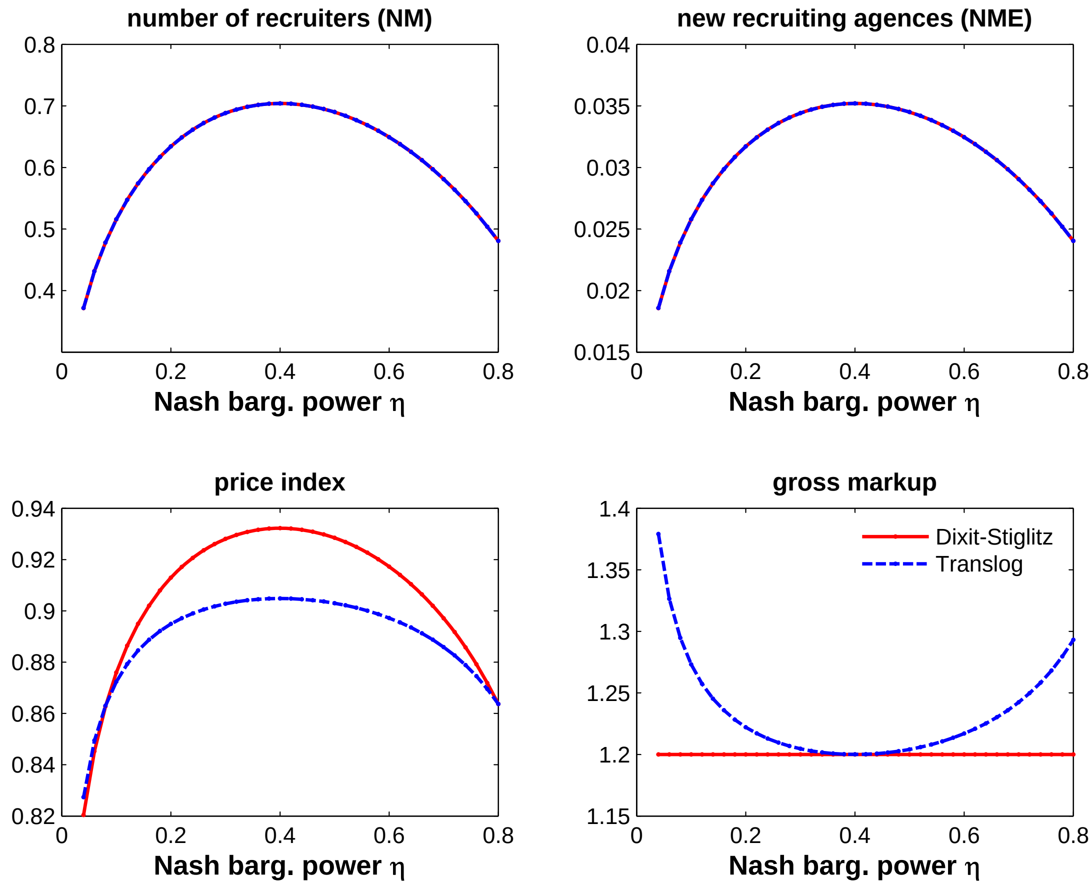

Figure 5: **Steady state as function of worker Nash wage bargaining power η in non-intermediated labor market III.** Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation. All other parameters besides η are held at their baseline values.

As the workers' bargaining power becomes increasingly higher, it becomes increasingly difficult to find employment in non-intermediated markets. There are two forces at play. First, as workers' bargaining power gets closer to "take-it-or-leave-it" offers (i.e., $\eta = 1$), this encourages potential new employees to continue to search in non-intermediated markets. Second, this same fact simultaneously leads to decreased job-finding probabilities as firms further reduce their non-intermediated market vacancies. This latter effect pushes households to start increasing their search in intermediated markets, ultimately leading to an increase in intermediated-market household searchers.

A similar rationale explains why vacancies in these markets initially increaseas the bargaining power of workers increases, firms decide to hire via a market where the bargaining power is not affecting how the employment surplus is splitbut for high levels of the bargaining power, the job-filling probabilities are increasingly influenced by the high measure of searchers, implying that firms dont need to post as many vacancies to generate a given number of matches. As a result, intermediated-market vacancies start to decline as the bargaining power of workers approaches 1. Finally, as the bargaining power of workers increases, unemployment increases, implying that recruiting firms find it less profitable to participate in matching markets, which ultimately leads to a decline in the number of recruiting firms.

4.3 Impulse Responses to a Positive TFP Shock

To analyze the response to a temporary shock to TFP, we follow the literature and consider an AR(1) process for TFP with persistence parameter 0.95 and a standard deviation of shocks equal to 0.007.[11]

Figure 6 and Figure 7 display impulse responses to a positive one-standard deviation TFP shock for, respectively, Dixit-Stiglitz aggregation in the recruiting sector and translog aggregation in the recruiting sector.

Baseline Economy. Consider a temporary increase in TFP under the baseline calibration of the model. An increase in TFP increases the marginal product of capital and labor and therefore pushes production firms to increase vacancy postings. Note that firms increase both intermediated and non-intermediated market vacancies, with the latter increasing by more than the former (i.e., vacancy postings in intermediated matching markets are more sensitive to TFP shocks than those taking place via intermediated matching markets). Intuitively, the non-intermediated matching market has lower steady-state matching efficiency. As a result, for a given positive shock to TFP and in relative terms, production firms gain comparably more by posting vacancies via this market compared to the intermediated market, which in turn explains why non-intermediated market vacancies increase by more. In response to the sharper increase in these vacancies, households redirect their searchers towards non-intermediated markets, resulting in an increase in searchers in the latter and a reduction in searchers in intermediated matching markets (not shown).

The above response in search behavior on the part of households explains the fact that the measure of new recruiters falls, leading to a reduction in active recruiters as well. Intuitively, while production firms continue to post vacancies in both matching markets, households redirect their search towards non-intermediated markets which, all else equal, lowers the effective probability of a match from the perspective of recruiting firms. In turn, this leads to a lower incentive in recruiting firm creation, despite the fact that production firms continue to post vacancies across matching markets amid temporarily higher TFP.

Finally, note that increased labor demand leads to higher labor force participation, which is driven by the increase in search for employment via non-intermediated matching markets. However,

[11] We log-linearize the model and implement a first-order approximation of the equilibrium conditions.

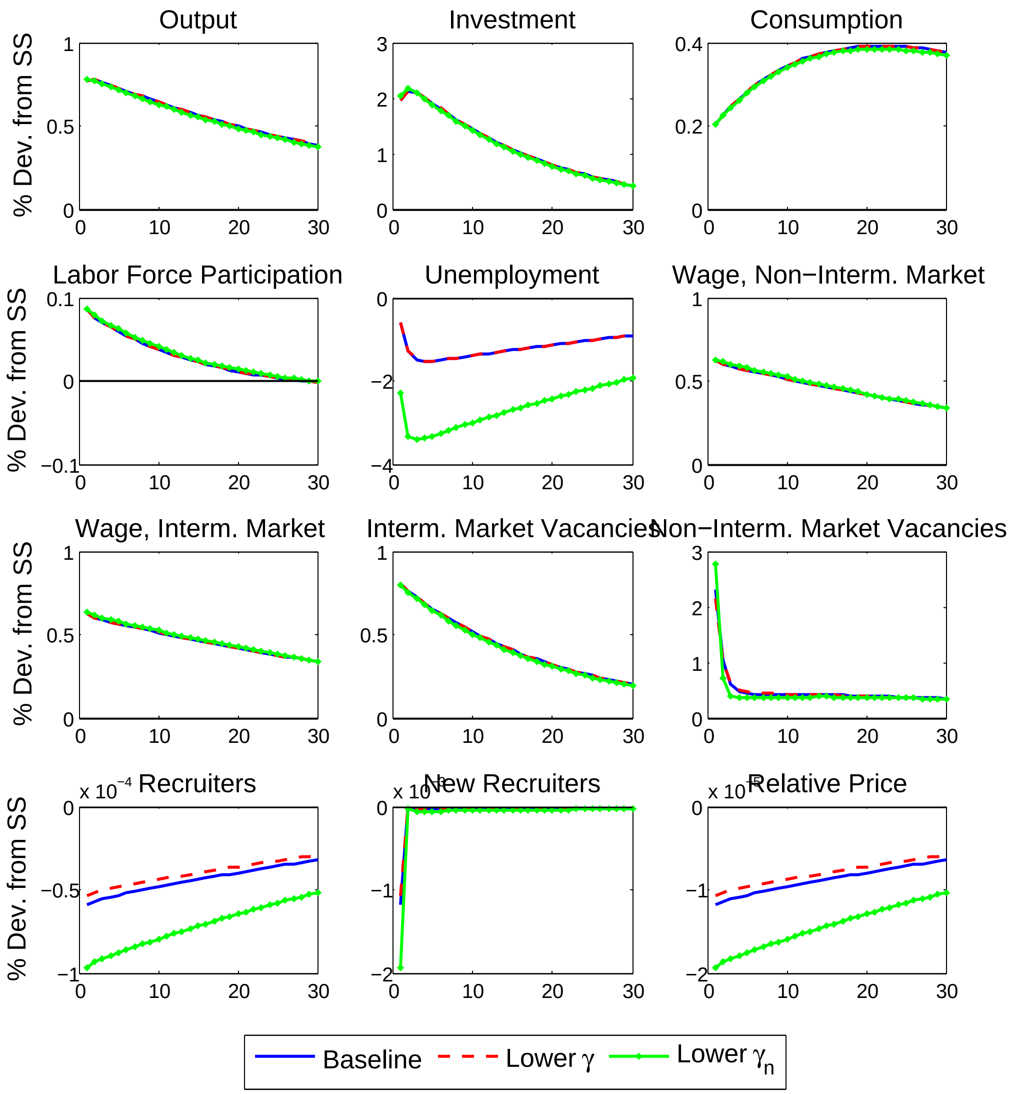

Figure 6: **Impulse Response Function to a Positive TFP Shock: Dixit-Stiglitz Aggregation.**

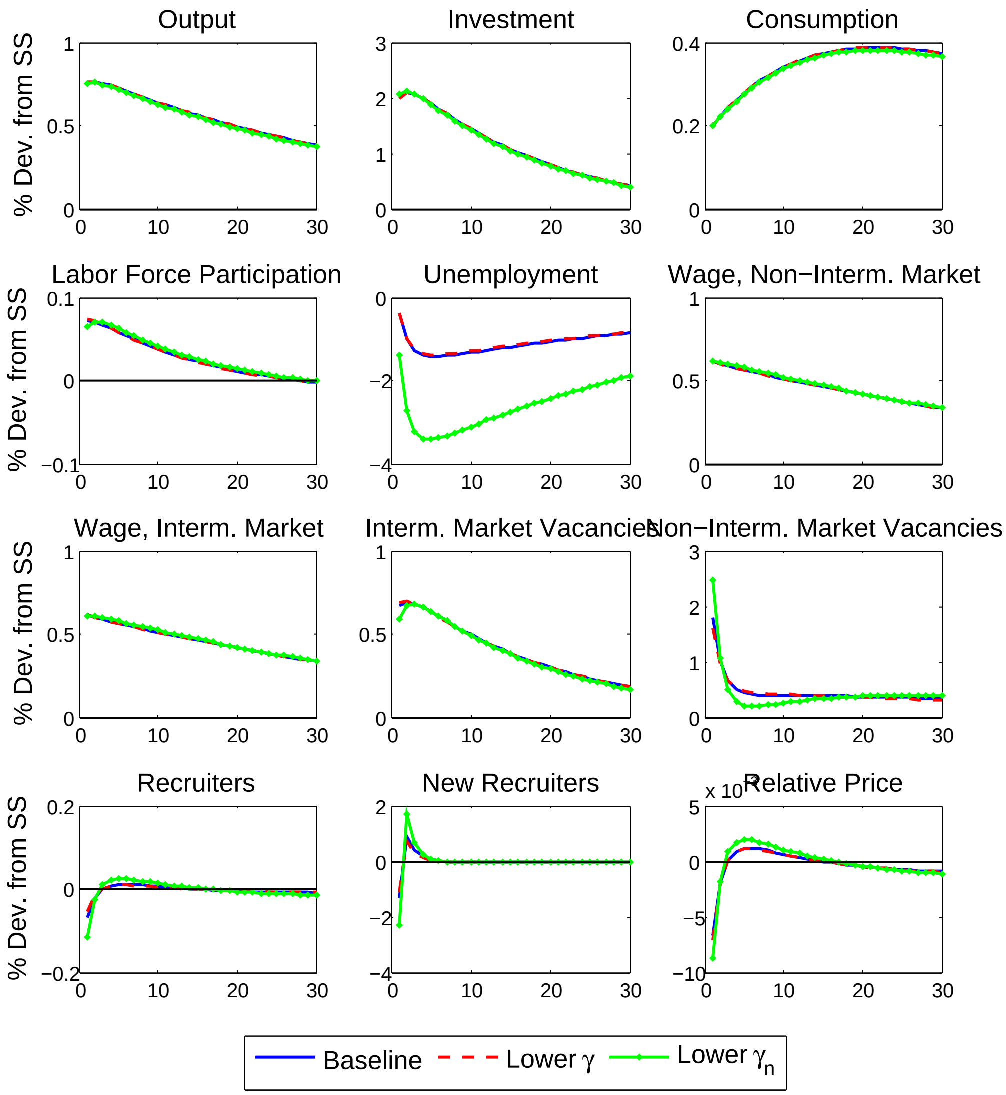

Figure 7: **Impulse Response Function to a Positive TFP Shock: Translog Aggregation.**

the boost in non-intermediated vacancy postings is larger than the increase in search activity, which ultimately leads to a reduction in unemployment. All told, positive TFP shocks lead to increases in GDP, investment, consumption, wages, and labor force participation, as well as lower unemployment and to a smaller number of recruiting firms (i.e., a more concentrated recruiting sector).

Lower Vacancy Posting Costs in Intermediated Matching Markets. Relative to the baseline economy, an economy with a lower cost of posting vacancies via intermediated matching markets (roughly two-thirds of the cost in the baseline economy) implies a very similar response in terms of macro aggregates. In fact, the only variables that exhibit a somewhat different response are the number of recruiting firms and new recruiters, as well as these firms' relative price. Quantitatively, though, the differences across economies are negligible. Intuitively, with a lower cost of posting vacancies, production firms respond less forcefully by posting vacancies in the non-intermediated matching market relative to the baseline economy. In turn, this implies that the intermediated matching market becomes somewhat more stable, leading to a smaller reduction in the equilibrium measure of recruiting firms. Of note, differences in steady-state equilibria across the two economies are very small. The most notable result is that, under lower vacancy posting costs in the intermediated matching market, unemployment is not lower and is instead marginally higher (6.19 percent vs. the baseline 6.17 percent).

Lower Vacancy Posting Costs in Non-Intermediated Matching Markets. Relative to the baseline economy, an economy with a lower cost of posting vacancies via non-intermediated matching markets (roughly two-thirds of the cost in the baseline economy) generates sharper responses in non-intermediated vacancy postings, non-intermediated searchers, new recruiting firms, and ultimately unemployment. Importantly, in contrast to the case with lower vacancy posting costs in intermediated matching markets, steady-state unemployment is considerably lower (the steady state unemployment rate is slightly higher than 2 percent, in contrast to the baseline of 6.17 percent). In turn, this partly explains why unemployment falls by more in response to the same positive TFP shock. Intuitively, a lower vacancy posting cost in non-intermediated matching markets lowers the expected marginal cost of posting vacancies, leading to higher steady-state vacancy postings, thereby making these vacancies more sensitive to TFP shocks. At the same time, households face higher steady-state wages in non-intermediated markets, and therefore a higher incentive to redirect their search towards these markets. The reduction in search behavior in the intermediated matching market increases the probability of finding a job for those who continue to search in that market (vacancies in that market still respond positively as a result of higher TFP), leading to higher job finding rates across the board relative to the baseline economy. Ultimately, both search behavior and vacancy postings in non-intermediated markets become more responsive, leading to a sharper decline in unemployment.

All told, a main message from this experiment is that not all vacancies are created equal. While intermediated matching markets have higher matching efficiency, their presence does not bring about lower labor market volatility and unemployment is as responsive as the baseline economy. Conversely, a reduction in the cost of posting vacancies in the non-intermediated matching market makes unemployment considerably more responsive.

5 Discussion and Relation to Literature

Existence of Middlemen.

One potential criticism of our model is that it does not endogenize the emergence of potentially "costly" labor-market intermediation. This criticism is somewhat misleading. There is no reason that *both* an intermediated labor market *and* a non-intermediated labor market cannot co-exist *as long as matching probabilities appropriately adjust* between intermediated and non-intermediated labor markets. Matching probabilities across intermediated and non-intermediated labor markets *do* adjust in our model.[12] As but one example in which probabilities do not appropriately adjust, suppose that, for unmodeled reasons outside the scope of this framework, wages in the intermediated sector are "rigid" over time. The wage rigidity would cause a failure in matching probabilities in the intermediated sector to appropriately adjust. In this case, it is clear that (as long as outcomes such as, say, labor rationing do not occur) the existence of the "middlemen" sector is a waste of resources and would therefore shut down.

Relation to Literature.

Relative to the existing literature on intermediaries — in particular, Masters (2007), Gautier, Hu, and Watanabe, and Farboodi, Jarosch, and Shimer (2017), among others — we stress our focus on market-structure imperfections in intermediated labor markets and endogenous entry among recruiting firms in the latter in a general equilibrium environment. Importantly, our framework emphasizes endogenous entry among intermediaries, whereas the existing literature has generally modeled whether individuals become intermediaries as opposed to producers. Given our interest in labor markets, it is natural to consider the creation of labor market intermediaries through the lens of firm creation. As a result, our modeling approachcentered on recruiting-firm entrycomplements existing theoretical work on middlemen and intermediaries.

More specifically, we put forth four main new results relative to existing work. First, the employment surplus between production firms and workers when matches take place via intermediated

[12] Note that "appropriate" adjustment is not synonymous with "efficient" adjustment.

markets is influenced by the competitiveness of the recruiting sector, with important implications for wages and therefore the incentive to search and post vacancies. Second, the presence of endogenous entry in the recruiting sector gives rise to increasing returns to scale in intermediated-based matching. While this is, in a broad sense, related to the environment in Masters (2007) where, under increasing returns, the matching rate is increasing in the number of people who participate in the market, our framework instead posits that the matching probabilities for production firms and workers in intermediated labor markets depends on the measure of intermediaries (i.e., recruiting firms) in addition to the number of individuals in the market (i.e., searchers and firm vacancies).[13] Then, the degree of increasing returns is intimately connected to (1) the cost of becoming a recruiting firm, and (2) the cost of posting vacancies in intermediated markets. This differs from the environment in Masters (2007). Third, focusing on a quantitative application, we stress that the behavior of intermediated labor markets is affected by the degree of efficiency in non-intermediated labor markets, with important implications for sectoral and overall labor market conditions (i.e. labor market tightness), unemployment, and participation. Finally, we show that changes in firms costs of attracting workers via vacancy posting can lead to widely different labor market outcomes, especially with respect to unemployment, depending on whether vacancy creation costs change in intermediated or non-intermediated labor markets

[13] Recall that the framework in Masters (2007) does not explicitly address labor markets, but his model can readily be applied to a labor market setting.

6 Conclusion

Labor market intermediaries are playing an increasingly relevant role in job matching. We introduce a monopolistically-competitive recruiting (intermediated) sector with endogenous entry into a general equilibrium search and matching model with non-intermediated labor markets to explore the labor market and aggregate implications of intermediated labor markets. Our framework features endogenous labor force participation, endogenous recruiting firm entry, a standard non-intermediated labor market, and production firms that use both capital and labor to produce. Focusing on the intermediated labor market, we show that surplus-sharing from employment relationships is directly influenced by the degree of competition in intermediary labor markets. Our framework features increasing returns to scale in intermediary-based matching. These two results imply that, in general equilibrium, recruiting firm profits have aggregate implications by modifying the absorption of production. Finally, we numerically show that both deviations from efficiency in non-intermediated markets and differential changes in the cost of posting vacancies across labor markets have important implications for the behavior of (long-run and cyclical) unemployment, thereby highlighting the relevance of understanding the behavior of intermediated labor markets for aggregate labor market outcomes.

Our framework is tractable enough to be used to explore several additional experiments, including the implications of an expanding recruiting sector for unemployment fluctuations, the role of differential changes in hiring costs across intermediated and non-intermediated markets for unemployment dynamics, and both labor market policy and optimal fiscal policy. We plan to explore these and other issues in future work.

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A Surplus Sharing

A.1 Envelope Condition with respect to s_{ijt}

Recall that recruiting firm ij's value function is given by

$$\mathbf{V}^M_{ij}(s_{ijt}, v_{ijt}; \cdot) = \rho_{Mijt} \cdot m(s_{ijt}, v_{ijt}) - mc_t \cdot m(s_{ijt}, v_{ijt}) - \Gamma_{Mt} N_{MEijt}. \tag{53}$$

Recruiting firm ij's envelope condition with respect to s_{ijt} is

$$\begin{aligned} \frac{\partial \mathbf{V}^M_{ij}(s_{ijt}, v_{ijt}; \cdot)}{\partial s_{ijt}} &= \rho_{Mijt} \cdot m_s(s_{ijt}, v_{ijt}) - mc_t \cdot m_s(s_{ijt}, v_{ijt}) \\ &= (\rho_{Mijt} - mc_t) \cdot \xi \cdot k^h(\theta_{ijt}), \end{aligned} \tag{54}$$

in which the second line uses the Cobb-Douglas matching function.[14] As per Moen (1997), recruiting firm ij chooses w_{ijt} and θ_{ijt} to optimize

$$\begin{aligned} &(\rho_{Mijt} - mc_t) \cdot \xi \cdot k^h(\theta_{ijt}) + \varphi^f_{ijt} \cdot \left[\gamma - k^f(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt})\right] \\ &+ \; 1 \cdot \left[k^h(\theta_{ijt}) \cdot \mathbf{W}(w_{ijt}) + (1 - k^h(\theta_{ijt})) \cdot \mathbf{U}_t - \mathbf{X}^H\right], \end{aligned} \tag{55}$$

with φ^f_{ijt} and 1 being the respective Lagrange multipliers on attracting vacancies towards submarket ij and on attracting actively searching individuals towards submarket ij.[15]

The first-order conditions with respect to w_{ijt} and θ_{ijt} are

$$-\varphi^f_{ijt} \cdot k^f(\theta_{ijt}) \cdot \frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}} + k^h(\theta_{ijt}) \cdot \frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}} = 0, \tag{56}$$

and

$$(\rho_{Mijt} - mc_t) \cdot \xi \cdot \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} - \varphi^f_{ijt} \cdot \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \cdot \mathbf{J}(w_{ijt}) + \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0. \tag{57}$$

Noting that $\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}} = -1$ and $\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}} = 1$ in our model, the multiplier φ^f_{ijt} is

$$\begin{aligned} \varphi^f_{ijt} &= -\frac{k^h(\theta_{ijt})}{k^f(\theta_{ijt})} \\ &= -\theta_{ijt}, \end{aligned} \tag{58}$$

[14] For ease of reference, the Cobb-Douglas matching function relationships are $m_s(s_{ijt}, v_{ijt}) = \xi \theta_{ijt}^{1-\xi}$, $k^f(\theta_{ijt}) = \theta_{ijt}^{-\xi}$, and $k^h(\theta_{ijt}) = \theta_{ijt}^{1-\xi}$.

[15] It is without of generality to normalize one of the multipliers due to the constant-returns matching function.

in which the second line follows due to Cobb-Douglas matching. Substituting φ^f_{ijt} in (57) gives

$$(\rho_{Mijt} - mc_t) \cdot \xi \cdot \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} + \theta_{ijt} \cdot \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \cdot \mathbf{J}(w_{ijt}) + \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0,$$

which, after substituting the Cobb-Douglas expressions $\frac{\partial k^h(\theta)}{\theta}$ and $\frac{\partial k^f(\theta)}{\theta}$ gives

$$(\rho_{Mijt} - mc_t) \cdot \xi \cdot (1-\xi)\theta_{ijt}^{-\xi} - \xi \theta_{ijt} \cdot \theta_{ijt}^{-\xi-1} \cdot \mathbf{J}(w_{ijt}) + (1-\xi)\theta_{ijt}^{-\xi} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0,$$

Dividing this expression by $(1-\xi)\,\theta_{ijt}^{-\xi}$ and slightly rearranging gives the surplus sharing rule

$$\xi \cdot (1-\xi) \cdot (\rho_{Mijt} - mc_t) + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{59}$$

If the matching aggregator were of Dixit-Stiglitz form, the surplus-sharing condition is

$$\xi \cdot (1-\xi) \cdot \frac{1}{\varepsilon} \underbrace{N_{Mijt}^{\frac{1}{\varepsilon-1}}}_{=\rho(N_{Mijt})} + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{60}$$

If the matching aggregator were of Benassy form, the surplus-sharing condition is

$$\xi \cdot (1-\xi) \cdot \frac{1}{\varepsilon} \underbrace{N_{Mijt}^{\varphi}}_{=\rho(N_{Mijt})} + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{61}$$

A.2 Envelope Condition with Respect to v_{ijt}

Recruiting firm ij's envelope condition with respect to v_{ijt} is

$$\begin{aligned}\frac{\partial \mathbf{V}_{ij}^{M}(s_{ijt}, v_{ijt}; \cdot)}{\partial v_{ijt}} &= \rho_{Mijt} \cdot m_v(s_{ijt}, v_{ijt}) - mc_t \cdot m_v(s_{ijt}, v_{ijt}) \\ &= (\rho_{Mijt} - mc_t) \cdot (1-\xi) \cdot k^f(\theta_{ijt}). \end{aligned} \tag{62}$$

As per Moen (1997), recruiting firm ij chooses w_{ijt} and θ_{ijt} to optimize

$$\begin{aligned}&(\rho_{Mijt} - mc_t) \cdot (1-\xi) \cdot k^f(\theta_{ijt}) + 1 \cdot \left[\gamma - k^f(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt})\right] \\ &+ \quad \varphi_{ijt}^h \cdot \left[k^h(\theta_{ijt}) \cdot \mathbf{W}(w_{ijt}) + (1 - k^h(\theta_{ijt})) \cdot \mathbf{U}_t - \mathbf{X}^H\right], \end{aligned} \tag{63}$$

with 1 and φ_{ijt}^h the respective Lagrange multipliers on attracting vacancies towards submarket ij and on attracting actively searching individuals towards submarket ij.[16]

The first-order conditions with respect to w_{ijt} and θ_{ijt} are

$$-k^f(\theta_{ijt}) \cdot \frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}} + \varphi_{ijt}^h \cdot k^h(\theta_{ijt}) \cdot \frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}} = 0 \tag{64}$$

and

$$(\rho_{Mijt} - mc_t) \cdot (1-\xi) \cdot \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} - \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \cdot \mathbf{J}(w_{ijt}) + \varphi_{ijt}^h \cdot \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0. \tag{65}$$

Noting that $\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}} = -1$ and $\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}} = 1$ in our model, the multiplier φ_{ijt}^h is

$$\begin{aligned}\varphi_{ijt}^h &= -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})} \\ &= -\theta_{ijt}^{-1}, \end{aligned} \tag{66}$$

in which the second line follows due to Cobb-Douglas matching.

Substituting φ_{ijt}^h in (65) gives

$$(\rho_{Mijt} - mc_t) \cdot (1-\xi) \cdot \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} - \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \cdot \mathbf{J}(w_{ijt}) - \theta_{ijt}^{-1} \cdot \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0,$$

which, after substituting the Cobb-Douglas expressions $\frac{\partial k^h(\theta)}{\theta}$ and $\frac{\partial k^f(\theta)}{\theta}$ gives

$$-(\rho_{Mijt} - mc_t) \cdot (1-\xi) \cdot \xi\theta_{ijt}^{-\xi-1} - \xi\theta_{ijt}^{-\xi-1} \cdot \mathbf{J}(w_{ijt}) + (1-\xi) \cdot \theta_{ijt}^{-1} \cdot \theta_{ijt}^{-\xi} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0.$$

[16]It is without of generality to normalize one of the multipliers due to the constant-returns matching function.

Dividing this expression by $(1-\xi)\,\theta_{ijt}^{-\xi-1}$ and slightly rearranging gives the surplus sharing rule

$$\xi \cdot (1-\xi) \cdot (\rho_{Mijt} - mc_t) + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{67}$$

If the matching aggregator were of Dixit-Stiglitz form, the surplus-sharing condition is

$$\xi \cdot (1-\xi) \cdot \frac{1}{\varepsilon} \underbrace{N_{Mijt}^{\frac{1}{\varepsilon-1}}}_{=\rho(N_{Mijt})} + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{68}$$

If the matching aggregator were of Benassy form, the surplus-sharing condition is

$$\xi \cdot (1-\xi) \cdot \frac{1}{\varepsilon} \underbrace{N_{Mijt}^{\varphi}}_{=\rho(N_{Mijt})} + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{69}$$

Also, as long as $\rho_{Mijt} > 0$, then the surplus sharing rule differs from the standard Nash surplus sharing rule in an environment where recruiting firms are not decentralized. In a perfectly competitive environment with zero costs of entry into the recruiting sector ($\Gamma_{Mt} = 0$), free entry dictates that the price of recruiting services ρ_{Mijt} should converge to the marginal cost of creating a new job match, suggesting that under perfect competition in recruiting and zero entry costs in the recruiting sector ($\Gamma_{Mt} = 0$), the Nash surplus sharing rule is the same as the one in an environment where there is no separate recruiting

B Firms

There is a continuum $[0,1]$ of identical goods-producing firms. The representative goods-producing firm's lifetime profit function is

$$E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left\{ z_t f(k_t, n_t) - r_t k_t - \gamma_N(v_t^N) - \int_0^1 \int_0^{N_{M,jt}} \gamma(v_{ijt}) \, di \, dj \right\}$$
$$- \quad E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left\{ w_t \cdot (1-\rho) n_{t-1} + w_t^N \cdot k_t^{fN} \cdot v_t^N + \int_0^1 \int_0^{N_{M,jt}} w_{ijt} \cdot k_{ijt}^f \cdot v_{ijt} \, di \, dj \right\} \quad (70)$$

subject to the period-t perceived law of motion of employment

$$n_t = (1-\rho) n_{t-1} + k_t^{fN} \cdot v_t^N + \int_0^1 \int_0^{N_{M,jt}} k_{ijt}^f \cdot v_{ijt} \, di \, dj. \quad (71)$$

Defining the Lagrange multiplier on the perceived law of motion (71) as μ_t, the first-order conditions with respect to k_t, v_{ijt}, v_t^N, and n_t are

$$z_t f_k(k_t, n_t) - r_t = 0, \quad (72)$$

$$\mu_t \cdot k_{ijt}^f - \gamma'(v_{ijt}) - w_{ijt} \cdot k_{ijt}^f = 0 \ \forall ij, \quad (73)$$

$$\mu_t \cdot k_t^{fN} - \gamma_N'(v_t^N) - w_t^N \cdot k_t^{fN} = 0, \quad (74)$$

and

$$-\mu_t + z_t f_n(k_t, n_t) + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\mu_{t+1} - w_{t+1} \right) \right\} = 0. \quad (75)$$

Isolating the multiplier μ_t from expression (74) gives

$$\mu_t = w_t^N + \frac{\gamma_N'(v_t^N)}{k_t^{fN}}, \quad (76)$$

and isolating the multiplier μ_t from expression (73) gives

$$\mu_t = w_{ijt} + \frac{\gamma'(v_{ijt})}{k_{ijt}^f} \ \forall ij. \quad (77)$$

Substituting the value for μ_t from (77) into (75) gives

$$\frac{\gamma'(v_{ijt})}{k_{ijt}^f} = z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma'(v_{jt+1})}{k_{jt+1}^f} + w_{jt+1} - w_{t+1} \right) \right\} \ \forall ij. \quad (78)$$

Next, substituting the value for μ_t from (76) into (75) gives

$$\frac{\gamma_N'(v_t^N)}{k_t^{fN}} = z_t f_n(k_t, n_t) - w_t^N + (1-\rho)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma_N'(v_{t+1}^N)}{k_{t+1}^{fN}} + w_{t+1}^N - w_{t+1}\right)\right\}. \tag{79}$$

B.1 Job-Creation Conditions

Without loss of generality, assuming that wages for incumbent employees in the periods after they were first hired (regardless of whether they were first hired through intermediated or non-intermediated labor markets) are identical simplifies the pair of expressions above to

$$\gamma'(v_{ijt}) = k_{ijt}^f \cdot \underbrace{\left(z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\gamma'(v_{jt+1})}{k_{jt+1}^f}\right\}\right)}_{\equiv \mathbf{J}(w_{ijt},\theta_{ijt})} \ \forall ij \tag{80}$$

and

$$\gamma_N'(v_t^N) = k_t^{fN} \cdot \underbrace{\left(z_t f_n(k_t, n_t) - w_t^N + (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\gamma_N'(v_{t+1}^N)}{k_{t+1}^{fN}}\right\}\right)}_{\equiv \mathbf{J}(w_t^N,\theta_t^N)}, \tag{81}$$

which characterize, respectively, costly job vacancies directed towards any intermediated labor submarket ij and costly job vacancies in the non-intermediated labor market. Around the optimum, the firm is indifferent between directing new job vacancies to intermediated submarket i or intermediated submarket k, $k^f(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}, \theta_{ijt}) = k^f(\theta_{kjt}) \cdot \mathbf{J}(w_{kjt}, \theta_{kjt})$, $\forall i \neq k$.

C Households

There is a continuum $[0,1]$ of identical households. In each household, there is a continuum $[0,1]$ of family members. In period t, each family member in the representative household has a labor-market status of employed, unemployed and actively seeking a job, or being outside the labor force. Regardless of which labor-market status a particular family member is in, each family member receives the same exact amount of consumption c_t due to full risk-sharing within each household (see Andolfatto (1996) for formal details).

The representative household maximizes lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\left(n_t + \underbrace{(1-k_t^{hN}) \cdot s_t^N}_{=ue_t^N} + \int_0^1 \left(\int_0^{N_{M,jt}} \underbrace{(1-k_{ijt}^h) \cdot s_{ijt}}_{=ue_{ijt}} \; di \right) dj \right) \right], \tag{82}$$

subject to the budget constraint

$$\begin{aligned} c_t + k_{t+1} + T_t = (1+r_t-\delta)k_t + w_t(1-\rho)n_{t-1} + w_t^N \cdot k_t^{hN} \cdot s_t^N + \int_0^1 \int_0^{N_{M,jt}} w_{ijt} \cdot k_{ijt}^h \cdot s_{ijt} \; di \; dj \\ + \quad (1-k_t^{hN}) \cdot s_t^N \chi + \int_0^1 \int_0^{N_{M,jt}} \left(1-k_{ijt}^h\right) \cdot s_{ijt}\chi \; di \; dj + \int_0^1 \Pi_{jt}^M \; di \cdot dj + \int_0^1 \Pi_{jt}^F \; di \cdot dj \end{aligned} \tag{83}$$

and the period-t perceived law of motion of employment

$$n_t = (1-\rho)n_{t-1} + k_t^{hN} \cdot s_t^N + \int_0^1 \int_0^{N_{M,jt}} k_{ijt}^h \cdot s_{ijt} \; di \; dj. \tag{84}$$

Defining the Lagrange multiplier on the flow budget constraint as λ_t and on the perceived law of motion as μ_t, the first-order conditions with respect to c_t, k_{t+1}, n_t, s_t^N, and s_{ijt} are

$$u'(c_t) - \lambda_t = 0, \tag{85}$$

$$-\lambda_t + \beta E_t \left\{ \lambda_{t+1} \left(1 + r_{t+1} - \delta\right) \right\} = 0, \tag{86}$$

$$-\mu_t - h'(lfp_t) + \beta(1-\rho) E_t \left\{ \lambda_{t+1} w_{t+1} + \mu_{t+1} \right\} = 0, \tag{87}$$

$$-(1-k_t^{hN}) \cdot h'(lfp_t) + \lambda_t \cdot \left(k_t^{hN} \cdot w_t^N + (1-k_t^{hN}) \cdot \chi \right) + \mu_t \cdot k_t^{hN} = 0, \tag{88}$$

and

$$-(1-k_{ijt}^h) \cdot h'(lfp_t) + \lambda_t \cdot \left(k_{ijt}^h \cdot w_{ijt} + (1-k_{ijt}^h) \cdot \chi \right) + \mu_t \cdot k_{ijt}^h = 0 \; \forall ij. \tag{89}$$

Isolating the multiplier μ_t from (88) gives

$$\frac{\mu_t}{u'(c_t)} = \left(\frac{1-k_t^{hN}}{k_t^{hN}}\right) \cdot \left(\frac{h'(lfp_t)}{u'(c_t)} - \chi\right) - w_t^N, \tag{90}$$

and isolating the multiplier μ_t from (89) gives

$$\frac{\mu_t}{u'(c_t)} = \left(\frac{1-k_{ijt}^{h}}{k_{ijt}^{h}}\right) \cdot \left(\frac{h'(lfp_t)}{u'(c_t)} - \chi\right) - w_{ijt} \ \forall ij, \tag{91}$$

in which both of these expressions have substituted the marginal utility of income $\lambda_t = u'(c_t)$ from (85).

Substituting the multiplier as stated in expression (90) into (87) yields

$$\begin{aligned}\left(\frac{1-k_t^{hN}}{k_t^{hN}}\right) \cdot \left(\frac{h'(lfp_t)}{u'(c_t)} - \chi\right) - w_t^N &= -\frac{h'(lfp_t)}{u'(c_t)} \\ &+ \ (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(w_{t+1} + \left(\frac{1-k_{t+1}^{hN}}{k_{t+1}^{hN}}\right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right) - w_{t+1}^N\right)\right\}.\end{aligned}$$

Cancelling the $-h'(lfp_t)/u'(c_t)$ terms and multiplying by k_t^{hN} gives

$$\begin{aligned}\frac{h'(lfp_t)}{u'(c_t)} &= k_t^{hN} w_t^N + (1-k_t^{hN})\chi \\ &+ \ k_t^{hN}(1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(w_{t+1} - w_{t+1}^N + \left(\frac{1-k_{t+1}^{hN}}{k_{t+1}^{hN}}\right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right)\right\}.\end{aligned}$$

Instead, substituting the multiplier as stated in expression (91) into (87) and following the same steps of algebra as above yields

$$\begin{aligned}\frac{h'(lfp_t)}{u'(c_t)} &= k_{ijt}^{h} w_{ijt} + (1-k_{ijt}^{h})\chi \\ &+ \ k_{ijt}^{h}(1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(w_{t+1} - w_{jt} + \left(\frac{1-k_{jt+1}^{h}}{k_{jt+1}^{h}}\right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right)\right\} \ \forall ij.\end{aligned}$$

C.1 Labor Force Participation Conditions

Without loss of generality, assuming that wages for incumbent employees in the periods after they were first hired (regardless of whether they were first hired through intermediated or non-intermediated labor markets) are identical simplifies the pair of expressions above to

$$\frac{h'(lfp_t)}{u'(c_t)} = k_t^{hN} \underbrace{\left[w_t^N + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left(\frac{1-k_{t+1}^{hN}}{k_{t+1}^{hN}} \right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \right]}_{\equiv \mathbf{W}(w_t^N, \theta_t^N)} + (1-k_t^{hN}) \underbrace{\chi}_{\equiv \mathbf{U}} \tag{92}$$

and

$$\frac{h'(lfp_t)}{u'(c_t)} = k_{ijt}^{h} \underbrace{\left[w_{ijt} + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left(\frac{1-k_{jt+1}^{h}}{k_{jt+1}^{h}} \right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \right]}_{\equiv \mathbf{W}(w_{ijt}, \theta_{ijt})} + (1-k_{ijt}^{h}) \underbrace{\chi}_{\equiv \mathbf{U}} \quad \forall ij, \tag{93}$$

which characterize, respectively, active job search in the non-intermediated labor market and active job search directed towards intermediated labor submarket ij. Given the household-level envelope conditions, around the optimum, active job search in all submarkets must yield the same value $k^h(\theta_{ijt}) \cdot \mathbf{W}(w_{ijt}, \theta_{ijt}) + \big(1 - k^h(\theta_{ijt})\big) \cdot \mathbf{U}(\cdot) = k^h(\theta_{kjt}) \cdot \mathbf{W}(w_{kjt}, \theta_{kjt}) + \big(1 - k^h(\theta_{kjt})\big) \cdot \mathbf{U}(\cdot), \forall i \neq k$.

D Derivation of Real Wage in Intermediated Market

Recall that the labor force participation condition can be written as

$$
\begin{aligned}
\frac{h'(lfp_t)}{u'(c_t)} &= k^h(\theta_{ijt})\left[w_{ijt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\mu_{jt+1}}{u'(c_{t+1})}\right\}\right] + (1-k^h(\theta_{ijt}))\cdot\chi \\
&= k^h(\theta_{ijt})\cdot \mathbf{W}(w_{ijt}) + (1-k^h(\theta_{ijt}))\cdot \mathbf{U}_t, \qquad (94)
\end{aligned}
$$

and

$$
\mathbf{W}(w_{ijt}) - \mathbf{U}_t = \frac{h'(lfp_t) - u'(c_t)\cdot\chi}{k^h(\theta_{ijt})\cdot u'(c_t)}. \qquad (95)
$$

In turn, the job creation condition is given by

$$
\frac{\gamma'(v_{ijt})}{k^f(\theta_{ijt})} = z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\gamma'(v_{jt+1})}{k^f(\theta_{jt+1})}\right\} = \mathbf{J}(w_{ijt}). \qquad (96)
$$

In recursive form, the surplus earned by the household is

$$
\mathbf{W}(w_{ijt}) - \mathbf{U}_t = w_{ijt} - \chi + (1-\rho)E_t\left\{\Xi_{t+1|t}\cdot(1-k^h(\theta_{jt+1}))\cdot(\mathbf{W}(w_{jt+1}) - \mathbf{U}_{t+1})\right\}, \qquad (97)
$$

and the surplus earned by the goods-producing firm is

$$
\mathbf{J}(w_{ijt}) = z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\mathbf{J}(w_{jt+1})\right\}. \qquad (98)
$$

Inserting expression (97) into the surplus-sharing condition

$$
\xi\cdot(\rho_{Mijt} - mc_t) + \mathbf{W}(w_{ijt}) - \mathbf{U}_t = \left(\frac{\xi}{1-\xi}\right)\cdot \mathbf{J}(w_{ijt}), \qquad (99)
$$

gives

$$
\begin{aligned}
&\xi\cdot(\rho_{Mijt} - mc_t) + w_{ijt} - \chi \\
&\quad + \quad (1-\rho)E_t\left\{\Xi_{t+1|t}\cdot(1-k^h(\theta_{jt+1}))\cdot(\mathbf{W}(w_{jt+1}) - \mathbf{U}_{t+1})\right\} = \left(\frac{\xi}{1-\xi}\right)\mathbf{J}(w_{ijt}). \qquad (100)
\end{aligned}
$$

Next, using the period-$t+1$ sharing rule gives

$$
\begin{aligned}
&\xi\cdot(\rho_{Mijt} - mc_t) + w_{ijt} - \chi \qquad (101) \\
&\quad + \quad (1-\rho)E_t\left\{\Xi_{t+1|t}\cdot(1-k^h(\theta_{jt+1}))\cdot\left[\left(\frac{\xi}{1-\xi}\right)\mathbf{J}(w_{jt+1}) - \xi\cdot(\rho_{Mjt+1} - mc_{t+1})\right]\right\} \\
&\quad = \quad \left(\frac{\xi}{1-\xi}\right)\mathbf{J}(w_{ijt}).
\end{aligned}
$$

Substituting $\mathbf{J}(w_{ijt}) = \frac{\gamma'(v_{ijt})}{k^f(\theta_{ijt})}$ and $\mathbf{J}(w_{ijt+1}) = \frac{\gamma'(v_{jt+1})}{k^f(\theta_{ijt+1})}$ yields

$$\begin{aligned} & \xi \cdot (\rho_{Mijt} - mc_t) + w_{ijt} - \chi \\ + \quad & (1-\rho)E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left[\left(\frac{\xi}{1-\xi} \right) \cdot \frac{\gamma'(v_{jt+1})}{k^f(\theta_{jt+1})} - \xi \cdot (\rho_{Mjt+1} - mc_{t+1}) \right] \right\} \\ = \quad & \left(\frac{\xi}{1-\xi} \right) \cdot \frac{\gamma'(v_{ijt})}{k^f(\theta_{ijt})}. \end{aligned} \tag{102}$$

Next, use the job-creation condition to substitute on the right-hand side, which gives

$$\begin{aligned} & \xi \cdot (\rho_{Mijt} - mc_t) + w_{ijt} - \chi \\ + \quad & (1-\rho)E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left[\left(\frac{\xi}{1-\xi} \right) \cdot \frac{\gamma'(v_{jt+1})}{k^f(\theta_{jt+1})} - \xi \cdot (\rho_{Mjt+1} - mc_{t+1}) \right] \right\} \\ = \quad & \left(\frac{\xi}{1-\xi} \right) \cdot \left(z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho)E_t \left\{ \Xi_{t+1|t} \frac{\gamma'(v_{jt+1})}{k^f(\theta_{jt+1})} \right\} \right). \end{aligned} \tag{103}$$

Grouping terms in w_{ijt},

$$\begin{aligned} & w_{ijt} \cdot \left(1 + \frac{\xi}{1-\xi} \right) = \left(\frac{\xi}{1-\xi} \right) z_t f_n(k_t, n_t) + \chi - \xi \cdot (\rho_{Mijt} - mc_t) \\ - \quad & (1-\rho)E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left[\left(\frac{\xi}{1-\xi} \right) \cdot \frac{\gamma'(v_{jt+1})}{k^f(\theta_{jt+1})} - \xi \cdot (\rho_{Mjt+1} - mc_{t+1}) \right] \right\} \\ + \quad & \left(\frac{\xi}{1-\xi} \right) \cdot (1-\rho)E_t \left\{ \Xi_{t+1|t} \frac{\gamma'(v_{jt+1})}{k^f(\theta_{jt+1})} \right\}. \end{aligned} \tag{104}$$

Rearranging,

$$\begin{aligned} & w_{ijt} \cdot \left(\frac{1}{1-\xi} \right) = \left(\frac{\xi}{1-\xi} \right) z_t f_n(k_t, n_t) + \chi - \xi \cdot (\rho_{Mijt} - mc_t) \\ - \quad & (1-\rho)E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left[\left(\frac{\xi}{1-\xi} \right) \cdot \frac{\gamma'(v_{jt+1})}{k^f(\theta_{jt+1})} - \xi \cdot (\rho_{Mjt+1} - mc_{t+1}) \right] \right\} \\ + \quad & \left(\frac{\xi}{1-\xi} \right) \cdot (1-\rho)E_t \left\{ \Xi_{t+1|t} \frac{\gamma'(v_{jt+1})}{k^f(\theta_{jt+1})} \right\}. \end{aligned} \tag{105}$$

Next, multiply by $(1-\xi)$, which gives

$$\begin{aligned} & w_{ijt} = \xi \cdot z_t f_n(k_t, n_t) + (1-\xi) \cdot \chi - (1-\xi) \cdot \xi \cdot (\rho_{Mijt} - mc_t) \\ - \quad & (1-\xi) \cdot (1-\rho)E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left[\left(\frac{\xi}{1-\xi} \right) \cdot \frac{\gamma'(v_{jt+1})}{k^f(\theta_{jt+1})} - \xi \cdot (\rho_{Mjt+1} - mc_{t+1}) \right] \right\} \\ + \quad & \xi \cdot (1-\rho)E_t \left\{ \Xi_{t+1|t} \frac{\gamma'(v_{jt+1})}{k^f(\theta_{jt+1})} \right\}. \end{aligned} \tag{106}$$

Expanding the terms that appear in the second line yields

$$\begin{aligned}
w_{ijt} &= \xi \cdot z_t f_n(k_t, n_t) + (1-\xi) \cdot \chi - (1-\xi) \cdot \xi \cdot (\rho_{Mijt} - mc_t) \\
&- \quad \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \frac{\gamma'(v_{jt+1})}{k^f(\theta_{jt+1})} \right\} \\
&+ \quad \xi \cdot (1-\xi) \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (\rho_{Mjt+1} - mc_{t+1}) \right\} \\
&+ \quad \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma'(v_{jt+1})}{k^f(\theta_{jt+1})} \right\}. \qquad (107)
\end{aligned}$$

Next, collect the terms that contain the monopolistic term $(\rho_{Mijt} - mc_t)$, which gives

$$\begin{aligned}
w_{ijt} &= \xi \cdot z_t f_n(k_t, n_t) + (1-\xi) \cdot \chi \\
&- \quad \xi \cdot (1-\xi) \cdot (\rho_{Mijt} - mc_t) + \xi \cdot (1-\xi) \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (\rho_{Mjt+1} - mc_{t+1}) \right\} \\
&- \quad \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \frac{\gamma'(v_{jt})}{k^f(\theta_{jt+1})} \right\} \\
&+ \quad \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma'(v_{jt})}{k^f(\theta_{jt+1})} \right\}. \qquad (108)
\end{aligned}$$

Expand the term in the third line yields

$$\begin{aligned}
w_{ijt} &= \xi \cdot z_t f_n(k_t, n_t) + (1-\xi) \cdot \chi \\
&- \quad \xi \cdot (1-\xi) \cdot (\rho_{Mijt} - mc_t) + \xi \cdot (1-\xi) \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (\rho_{Mjt+1} - mc_{t+1}) \right\} \\
&- \quad \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma'(v_{jt+1})}{k^f(\theta_{ijt+1})} \right\} + \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma'(v_{jt+1}) \cdot k^h(\theta_{jt+1})}{k^f(\theta_{jt+1})} \right\} \\
&+ \quad \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma'(v_{jt+1})}{k^f(\theta_{jt+1})} \right\}. \qquad (109)
\end{aligned}$$

After cancelling terms in the third and fourth lines and using the Cobb-Douglas functional form $\frac{k^h(\theta)}{k^f(\theta)} = \theta$, the submarket ij wage is

$$\begin{aligned}
w_{ijt} &= \xi z_t f_n(k_t, n_t) + (1-\xi)\chi + \xi(1-\rho) E_t \left\{ \Xi_{t+1|t} \gamma'(v_{jt+1}) \theta_{jt+1} \right\} \qquad (110) \\
&- \quad \xi(1-\xi) \left(\rho_{Mijt} - mc_t \right) + \xi(1-\xi)(1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\rho_{Mjt+1} - mc_{t+1} \right) \right\}.
\end{aligned}$$

E Aggregation

The (symmetric equilibrium) flow budget constraint of the government is

$$T_t = g_t(1 - k^h(\theta_t)) \cdot s_t \cdot N_{Mt} \cdot \chi + (1 - k^{hN}(\theta_t^N)) \cdot s_t^N \cdot \chi, \tag{111}$$

in which lump-sum taxes T_t levied on households finance government-provided unemployment benefits and government spending g_t.

E.1 Aggregate Goods Resource Constraint

To construct the aggregate symmetric equilibrium household budget constraint, begin with expression (83), which is repeated here for convenience:

$$\begin{aligned} c_t + k_{t+1} + T_t = {} & (1 + r_t - \delta)k_t + w_t(1-\rho)n_{t-1} + w_t^N \cdot k_t^{hN} \cdot s_t^N + \int_0^1 \int_0^{N_{Mjt}} w_{ijt} \cdot k_{ijt}^h \cdot s_{ijt} \; di \; dj \\ + & \quad (1 - k_t^{hN}) \cdot s_t^N \chi + \int_0^1 \int_0^{N_{Mjt}} \left(1 - k_{ijt}^h\right) \cdot s_{ijt}\chi \; di \; dj + \int_0^1 \Pi_{jt}^M \; di \cdot dj + \int_0^1 \Pi_{jt}^F \; di \cdot dj \end{aligned} \tag{112}$$

Integrating over the i intermediated submarkets in each labor market j gives

$$\begin{aligned} c_t + k_{t+1} + T_t = {} & (1 + r_t - \delta)k_t + w_t(1-\rho)n_{t-1} + w_t^N \cdot k_t^{hN} \cdot s_t^N + \int_0^1 N_{Mjt} \cdot w_{jt} \cdot k_{jt}^h \cdot s_{jt} \; dj \\ + & \quad (1 - k_t^{hN}) \cdot s_t^N \chi + \int_0^1 N_{Mjt} \cdot \left(1 - k_{jt}^h\right) \cdot s_{jt}\chi \; dj + \int_0^1 \Pi_{jt}^M \; dj + \int_0^1 \Pi_{jt}^F \; dj. \end{aligned}$$

Next, integrating over the measure $j \in (0, 1)$ of recruiting markets gives the symmetric equilibrium household budget constraint

$$\begin{aligned} c_t + k_{t+1} + T_t = {} & (1 + r_t - \delta)k_t + w_t(1-\rho)n_{t-1} + w_t^N \cdot k_t^{hN} \cdot s_t^N + N_{Mt} \cdot w_t \cdot k_t^h \cdot s_t \\ + & \quad (1 - k_t^{hN}) \cdot s_t^N \chi + N_{Mt} \cdot \left(1 - k_t^h\right) \cdot s_t\chi + \Pi_t^M + \Pi_t^F. \end{aligned}$$

Combining this with with the government budget (111) gives

$$c_t + k_{t+1} + (1-\delta)k_t = w_t(1-\rho)n_{t-1} + w_t^N \cdot k_t^{hN} \cdot s_t^N + N_{Mt} \cdot w_t \cdot k_t^h \cdot s_t + r_t k_t + \Pi_t^M + \Pi_t^F. \tag{113}$$

In symmetric equilibrium, the period-t flow profits Π_t^F are

$$\Pi_t^F = z_t f(k_t, n_t) - w_t(1-\rho)n_{t-1} - w_t^N \cdot k_t^{hN} \cdot s_t^N - N_{Mt} \cdot w_t \cdot k_t^h \cdot s_t - r_t k_t - \gamma(v_t) \cdot N_{Mt} - \gamma_N(v_t^N) \tag{114}$$

and aggregate recruiting-firm profits Π_t^M are

$$\begin{aligned}\Pi_t^M &= [\rho_{Mt} \cdot m(s_t, v_t) - mc_t \cdot m(s_t, v_t)] \cdot N_{Mt} - \Gamma_{Mt} N_{MEt} \\ &= (\rho_{Mt} - mc_t) \cdot m(s_t, v_t) \cdot N_{Mt} - \Gamma_{Mt} N_{MEt}.\end{aligned}$$

Substituting Π_t^F and Π_t^M into (113) gives the decentralized economy's aggregate goods resource constraint

$$\begin{aligned}& c_t + k_{t+1} - (1-\delta)k_t + \gamma(v_t) \cdot N_{Mt} \\ + \quad & \gamma_N(v_t^N) + \Gamma_{Mt} N_{MEt} - (\rho_{Mt} - mc_t) \cdot N_{Mt} \cdot m(s_t, v_t) = z_t f(k_t, n_t). \end{aligned} \tag{115}$$

Several novel features of the aggregate goods resource constraint (115) are worth emphasizing.

1. The absorption of resources devoted to the recruiting industry, $(\rho_{Mt} - mc_t) \cdot N_{Mt} \cdot m(s_t, v_t)$.
2. The resources devoted to the recruiting industry diminish as $(\rho_{Mt} - mc_t) \to 0$, in which case the *perfectly*-competitive search equilibrium described by Moen (1997) emerges.
3. The appearance of the increasing returns to scale that emerges from the differentiated recruiting sector, captured in the term $N_{Mt} \cdot m(s_t, v_t)$.

E.2 Private-Sector Equilibrium

A symmetric private-sector general equilibrium is made up of **sixteen** endogenous state-contingent processes $\{c_t, n_t, lfp_t, k_{t+1}, N_{Mt}, N_{MEt}, s_t, v_t, \theta_t, w_t, s_t^N, v_t^N, \theta_t^N, w_t^N, \rho_{Mt}, mc_t\}_{t=0}^{\infty}$ that satisfy the following sixteen sequences of conditions: the aggregate resource constraint

$$\begin{aligned}& c_t + k_{t+1} - (1-\delta)k_t + \gamma(v_t) \cdot N_{Mt} \\ + \quad & \gamma_N(v_t^N) + \Gamma_{Mt} N_{MEt} - (\rho_{Mt} - mc_t) \cdot N_{Mt} \cdot m(s_t, v_t) = z_t f(k_t, n_t), \end{aligned} \tag{116}$$

the aggregate law of motion for labor

$$n_t = (1-\rho)n_{t-1} + m(s_t^N, v_t^N) + N_{Mt} \cdot m(s_t, v_t), \tag{117}$$

the definition of aggregate LFP

$$lfp_t = N_t + (1 - k_t^{hN})s_t^N + (1 - k_t^h)s_t, \tag{118}$$

the aggregate law of motion for recruiters

$$N_{Mt} = (1-\omega)N_{M,t-1} + N_{MEt}, \tag{119}$$

the capital Euler condition

$$1 = E_t\left\{\Xi_{t+1|t}\left(1 + z_{t+1}f_k(k_{t+1}, n_{t+1}) - \delta\right)\right\}, \tag{120}$$

the free-entry condition for recruiters

$$\Gamma_{Mt} = (\rho_{Mt} - mc_t)\, m(s_t, v_t) + (1-\omega)E_t\left\{\Xi_{t+1|t}\Gamma_{Mt+1}\right\}, \tag{121}$$

the vacancy creation condition for intermediated labor markets

$$\gamma'(v_{ijt}) = k^f(\theta_t)\cdot \underbrace{\left(z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\gamma'(v_{jt+1})}{k^f_{jt+1}}\right\}\right)}_{\equiv \mathbf{J}(w_{ijt}, \theta_{ijt})} \;\forall ij, \tag{122}$$

the vacancy creation condition for non-intermediated labor markets

$$\gamma'_N(v^N_t) = k^{fN}(\theta^N_t)\cdot \underbrace{\left(z_t f_n(k_t, n_t) - w^N_t + (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\gamma'_N(v^N_{t+1})}{k^{fN}_{t+1}}\right\}\right)}_{\equiv \mathbf{J}(w^N_t, \theta^N_t)}, \tag{123}$$

the active job search condition for non-intermediated labor markets

$$\frac{h'(lfp_t)}{u'(c_t)} = k^{hN}_t \underbrace{\left[w^N_t + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(\frac{1-k^{hN}_{t+1}}{k^{hN}_{t+1}}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\}\right]}_{\equiv \mathbf{W}(w^N_t, \theta^N_t)} + (1-k^{hN}_t)\underbrace{\chi}_{\equiv \mathbf{U}}, \tag{124}$$

the active job search condition directed towards intermediated labor markets

$$\frac{h'(lfp_t)}{u'(c_t)} = k^h_{ijt} \underbrace{\left[w_{ijt} + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(\frac{1-k^h_{jt+1}}{k^h_{jt+1}}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\}\right]}_{\equiv \mathbf{W}(w_{ijt}, \theta_{ijt})} + (1-k^h_{ijt})\underbrace{\chi}_{\equiv \mathbf{U}} \;\forall ij, \tag{125}$$

the surplus-sharing rule that determines wages w_t in monopolistic labor markets

$$\xi\cdot(\rho_{Mt} - mc_t) + \mathbf{W}(w_t) - \mathbf{U}_t = \left(\frac{\xi}{1-\xi}\right)\mathbf{J}(w_t), \tag{126}$$

the surplus-sharing rule that determines Nash-bargained wages (with η denoting the employee's Nash bargaining power) in non-intermediated labor markets

$$\mathbf{W}(w_t^N) - \mathbf{U}_t = \left(\frac{\eta}{1-\eta}\right)\mathbf{J}(w_t^N), \tag{127}$$

the monopolistic matching-market pricing expression

$$\rho(N_{Mt}) = \mu(N_{Mt}) \cdot mc(N_{Mt}), \tag{128}$$

the definition of labor-market tightness in monopolistic recruiting markets

$$\theta_t = \frac{v_t}{s_t}, \tag{129}$$

the definition of labor-market tightness in monopolistic recruiting markets

$$\theta_t^N = \frac{v_t^N}{s_t^N}, \tag{130}$$

and the symmetric equilibrium ρ_{Mt}

$$\rho_{Mt} = \rho(N_{Mt}), \tag{131}$$

which depends on the particular matching aggregator.