Learning on the Job and the Cost of Business Cycles*

Preliminary version - please do not quote

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Abstract

We show that business cycle variation reduces welfare through a decrease in the average level of employment and output in a labor market search model with learning on-the-job. The key mechanism is the following: It is well established that the Beveridge correlation is negative, i.e. that vacancies and unemployment are negatively correlated. Via the matching function business cycles therefore tend to reduce the average number of new jobs and, in turn, employment. Then, since learning on-the-job imply that aggregate human capital is increasing in employment, it follows that aggregate volatility reduces human capital. This, in turn, reduces the incentives to post vacancies, further reducing employment and human capital. We quantify this mechanism using a carefully calibrated model and find the output and welfare cost of business cycles to be large.

Keywords: Search and matching, earnings loss, labor market, human capital, stabilization policy.

JEL classification: E32, J64.

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1 Introduction

A major question in macroeconomics is whether welfare costs of business cycles are substantial or not. Since Lucas (1987) it has been well established that the cost of aggregate consumption fluctuations is negligible. Business cycles can induce welfare costs in other ways though, e.g. through their effect on the cross-sectional distribution of consumption (Imrohoroğlu, 1989, and many others). Furthermore, business cycles may affect welfare through the average level of output, an issue that has been pushed recently by Summers (2015) who emphasizes the importance of stabilization policy to counteract this. One recent example of a model with this type of mechanism is Hassan and Mertens (2017), which show that expectational errors about future productivity increase risk-premia and reduce output. Our paper adds to this literature by presenting a new mechanism for how business cycles, or more generally aggregate volatility, reduce the level of output.

We show that business cycles substantially reduce the level of employment, output and welfare in a labor market search model with learning on-the-job. The key mechanism of the paper is the following: It is well established that the Beveridge correlation is negative, i.e. that vacancies and unemployment are negatively correlated in the data (see e.g. Fujita and Ramey, 2012). Via the matching function, this implies that business cycles tend to reduce the average number of new jobs and hence employment.[1] Then, since learning on-the-job implies that average human capital is increasing in employment, it follows that aggregate volatility reduces human capital. This, in turn, reduces the incentives to post vacancies, further reducing employment and so on in a vicious circle. This mechanism for how aggregate volatility reduces employment, human capital and thereby output is illustrated graphically in Figure 1. The size of the cost of business cycles generated by this mechanism is accordingly largely determined by how sensitive the human capital distribution is to changes in employment, and how sensitive job creation is to changes in the human capital distribution.

We have here emphasized the correlation between vacancies and unemployment as driving the negative effect of aggregate volatility on the average level of employment because this is the main

[1] A negative Beveridge correlation implies that $cov\left(v,u\right)<0$ with v denoting vacancies and u unemployment. Jung and Kuester (2011) states conditions for when aggregate volatility implies a reduction of employment in a simple search and matching model. Note that, using a first-order approximation,

$$cov\left(v,u\right)=\frac{1}{1-\omega}cov\left(f,u\right)+var\left(u\right)$$

where ω is the matching function elasticity. Then, using the employment flow equation $1-u_t=\left(1-\delta\right)\left(1-u_{t-1}\right)+f_{t-1}u_{t-1}$ with δ denoting the exogenous separation rate, we have, proceeding along the lines of Jung and Kuester (2011),

$$Eu-u=-\frac{1}{\delta}\underbrace{\left(1-\omega\right)\left(cov\left(v,u\right)-var\left(u\right)\right)+\left(Ef-f\right)Eu+\left(Eu-u\right)f}_{Em-m}.$$

As can be seen from this expression, unemployment under aggregate volatility is higher if the Beveridge correlation is negative and $Ef\leq f$.

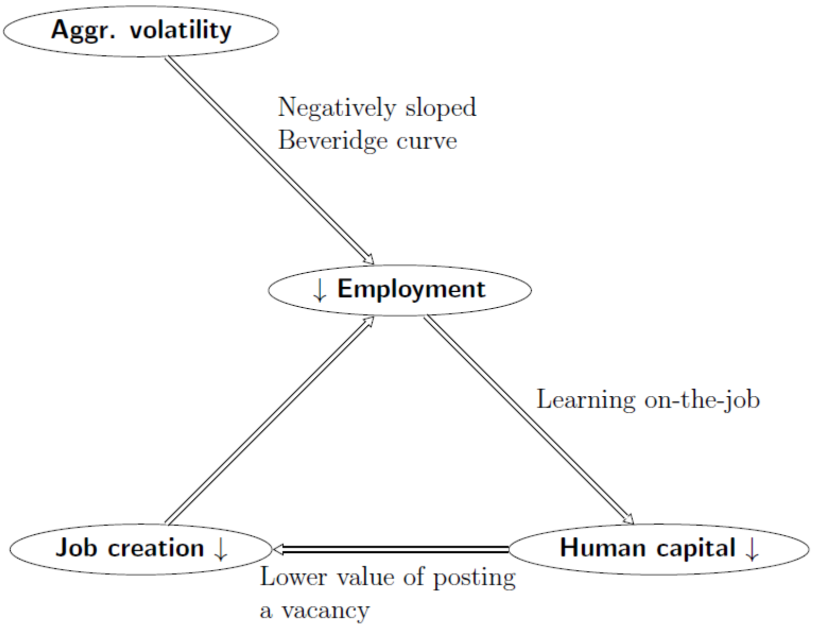

Figure 1: Illustration of main mechanism - how aggregate volatility reduces employment, human capital and thereby output.

factor in our model. [2] Regardless of the source, any reduction in average employment is amplified by the positive relationship between the human capital distribution and employment induced by learning on-the-job. This extends beyond the costs of business cycles. For example, the effect of a change in taxation that changes the average employment level will be amplified by the human capital mechanism that we have outlined.

Another indication that business cycles reduce earnings, and hence output, comes from the literature on earnings losses from job displacement. Davis and von Wachter (2011, DvW henceforth) document empirically that both the frequency of job displacement and the present discounted earnings losses per displaced worker are increasing in the unemployment rate.[3] Together, these two facts imply that job displacement occurs at a higher frequency at times when it is more costly. An economy with the same average displacement rate but without any unemployment volatility would yield lower aggregate earnings losses due to displacement. These earnings losses due to job displacement constitute an overlooked potential component of the welfare costs of business cycles.

We model the above phenomenon in a general equilibrium framework with a search and matching

[2] More generally, any convex cost (or concave benefit or production function) in any cyclical variable tend to induce a negative relationship between aggregate volatility and employment. A prominent example is convex capital adjustment costs, which is commonly assumed in the business cycle literature.

[3] Similar empirical results are obtained in Jacobson et al (1993) and Farber (2005).

labor market. An important goal when setting up our model is to provide a credible quantification of the cost of business cycles through the mechanism we have sketched above. A key determinant of these costs is the speed of human capital loss during unemployment. Earnings losses of displaced workers is informative about this. DvW showed that earnings losses in the textbook search and matching model is substantially smaller than what is empirically observed. Adding only human capital dynamics to the classical search and matching model does not give rise to earnings losses of the right magnitude since workers find new jobs quickly.[4] In addition to human capital dynamics, we choose to allow for an additional mechanism that drives earnings losses in the direction of the empirical evidence. This mechanism is a job ladder, stemming from idiosyncratic match productivity and on-the-job search. Both unemployed and employed workers search for jobs, implying that worker wages then depend on job offers from other firms, i.e. outside options, giving rise to a wage ladder. This implies that a worker builds up "negotiation capital" when continuously employed and hence get an increasing share of the surplus. When loosing the job, the reemployment wage of the worker then decreases more than the fall in productivity.

The model is therefore quite rich in some dimensions. It accounts for heterogeneity on both the firm and worker side. Firms differ in match quality which is subject to shocks. Workers differ in the level of general human capital which is determined by learning on-the-job, i.e. that workers gain human capital while employed, and lose human capital when unemployed. Direct evidence of such human capital dynamics is provided by Edin and Gustavsson (2008). This type of dynamics has also been documented to be important for generating persistent earnings losses from job displacement (see Burdett, Carrillo-Tudela and Coles, 2015, Huckfeldt, 2016, Jarosch, 2015).

We calibrate our model by matching a large number of moments, including volatility of GDP and unemployment, standard worker flow moments, the degree of wage dispersion, the cyclicality of job separations, and earnings losses per worker. We then compute the cost of business cycles by comparing the results for our full model to the results from the same model, but without aggregate volatility.[5] We find that business cycles reduce steady state employment, GDP and welfare by substantial amounts. In particular, welfare (GDP) fall by 2.7-4.1 percent (4.1 percent) due to aggregate volatility. These are large effects. Accounting for the transition dynamics the welfare gains of eliminating business cycles are slightly smaller, 1.5-2.6 percent. Human capital dynamics in the form of learning on-the-job are pivotal for the results - if we disable them in our model the implied GDP and welfare losses from business cycles are substantially reduced.

[4] To get reasonable earnings losses additional mechanisms need to be added, see for example Huckfeldt (2016) and Jarosch (2015).

[5] Note that we assume risk-neutrality for computational reasons. This implies that we do not capture the full welfare cost of business cycles. Our results can accordingly be interpreted as a lower bound for the cost of business cycles.

There is indicative empirical support for the relationship between aggregate volatility, unemployment and output implied by our model. Most closely related to our mechanism, there is a small literature regarding the relationship between volatility and the unemployment rate. Hairault et al. (2010) use data for 20 OECD countries for the period 1982-2003 and find significant positive effects of TFP volatility on average unemployment. More generally, there is ample evidence of a significant negative relationship between volatility of output and the growth rate of output. The seminal paper establishing this cross-country relationship is Ramey and Ramey (1995). Luo et al. (2016) confirm this result using more recent data.

The welfare effects of the mechanism we are documenting works through the average level of output, or, to be exact, consumption. In this sense it is fundamentally different from most of the cost of business cycles literature, which analyses the effects of business cycles on welfare through (aggregate or idiosyncratic) consumption volatility, including papers that, like ours, model idiosyncratic countercyclical labor income risk related to job displacement, but where this idiosyncratic risk is the costly part of business cycles (Krebs (2007) and Berger et al (2016)).[6]

Three papers have previously emphasized the effect of business cycles on the average level of output in a search and matching labor market setting. Den Haan and Sedlacek (2014) quantified the cost of business cycles in a setting where an agency problem generates inefficient job separations in downturns thereby reducing employment and GDP. Our framework does not include any such agency problem. In fact, the role of market imperfections for the cost of business cycles is negligible in our setting. Jung and Kuester (2011) derived and quantified the effects on employment and welfare of the negative correlation between the job finding rate and the unemployment rate. They did so in a simpler setting than ours, using a solution method of local second-order approximations, with wages assumed to be independent of tightness, and without endogenous separations.[7] Hairault et al (2010) also look at this issue in a setting without human capital dynamics. Both papers find that the business cycle effect on GDP and welfare is well below one percent, i.e. on an order of magnitude smaller than ours.

In terms of modelling, if not research question, our paper is closely related to Lise and Robin (2017), henceforth LR. As LR, we use global solution methods to solve the model. Our model also shares mechanisms with a number of papers that analyzes earnings losses from displacement (Burdett, Carrillo-Tudela and Coles, 2015, Huckfeldt, 2016, Jarosch, 2015, Jung and Kuhn, 2016, and Krolikowski, 2015). All of these papers, except Huckfeldt (2016), abstract from aggregate volatility.

[6] It is also distinct from the mechanism in Krebs (2003). There countercyclical idiosyncratic income risk imply that business cycles reduce human capital investment, and for low values of risk aversion, total (physical + human) investment.

[7] In an extension they allowed for learning on-the-job, but assumed a weaker dependence of human capital on employment than we do.

The paper is outlined as follows: Section 2 presents the model, Section 3 documents the calibration and Section 4 provides the quantitative results. Finally, Section 5 concludes.

2 Model

The basic building blocks of our model are similar to LR.[8] In terms of human capital dynamics the model is in the tradition of Pissarides (1992) and Ljungqvist and Sargent (1998). As in these papers, we model general human capital as stemming from learning on-the-job. We include heterogeneity both on the worker and the firm side. Worker human capital is indexed by x and the match-specific productivity is indexed by y. Both x and y follow stochastic processes. Let the Markov transition probability $\pi_y(y, y')$ denote the dynamics of the match-specific productivity and let $\pi_{xe}(x, x')$ ($\pi_{xu}(x, x')$) denote the Markov transition probability for the worker's human capital level while employed (unemployed). Human capital of employed workers is weakly increasing while for unemployed workers it is weakly decreasing, reflecting an assumption of learning on-the-job.[9]

Each firm employs (at most) one worker and output from a match is $p(x, y, z) = xyz$ where z denotes aggregate TFP and $\pi(z, z')$ denotes the Markov transition probability of z. There is no physical capital. Workers search for jobs both when employed and unemployed. Utility is linear in consumption and the discount factor is β. Wages are determined by Bertrand competition between firms so that a worker always receive a value equal to his outside option. This determination of wages follows Postel-Vinay and Robin (2002) and LR and is generally referred to as "sequential auctions". Finally, in order to capture human capital dynamics, workers die with probability ν.

Let us here mention two computational aspects of the model which is solved using global solution methods. First, wage determination through sequential auctions jointly with risk neutrality and a common discount factor implies that the expected value of a match for an employer (and hence the value of posting a vacancy) is independent of the future cross-sectional distributions of workers and firms. This was pointed out by LR and simplifies computations significantly. In particular, equilibrium allocations can be solved for without computing the expected next period distribution of workers across

[8] Compared to LR the features we add are i) Accumulation of human capital, x, on-the-job as well as decumulation during unemployment, and ii) idiosyncratic shocks to the match-specific productivity y. These assumptions are made to generate empirically relevant persistence of earnings losses and separations, respectively. Another difference from LR is that in our model the productivity y of a match is not known when a vacancy is posted. This last difference substantially simplifies the computation of individual wages. LR did not compute wages.

[9] Our human capital dynamics differ from Ljungqvist and Sargent (1998, 2008) and Jung and Kuester's (2011) extension with human capital. They did not model heterogeneity in match-specific productivity and presumably therefore assumed, as a short-cut, that part of the human capital loss occur when a worker is separated from a job and the remaining part occurs gradually during unemployment. With only exogenous separations this reduces the dependence of the human capital distribution on employment (or any endogenous variable in the model).

firms and within the pool of unemployed. Second, computing individual wages and value functions for workers are still non-trivial tasks, because current wages depend on the probability of receiving a job offer the next period. This, in turn, depends on the next period's labor market tightness. A key determinant for next period's tightness is the expected value to a firm of matching with a worker in the next period, which in turn depends on the next period distribution of workers and firms. Fortunately, the equilibrium conditions of our model indicate three moments that fully capture the implications of this large dimensional object. We then use a Krusell and Smith (1998) style algorithm to let these three moments summarize and predict the labor market tightness, thereby enabling us to solve for the wages. For details on the solution algorithm see Appendix A.2.

2.1 Preliminaries

Let us start the detailed model description by providing an overview of the timing protocol. The sequence of events within a period are the following: First, the aggregate productivity shock z and the idiosyncratic shocks (x, y) are realized. Second, the fraction ν of workers that die is replaced by newborn unemployed workers with human capital at the lowest possible level, $\underline{x}$. Third, separations into unemployment occur. Then firms post vacancies and workers search. Finally, new matches are formed, wages are set and production takes place.

Let $1\{\}$ denote the indicator function and let $S(x, y, z)$ denote the total surplus of a match. Matches with negative total surplus $S(x, y, z)$ are endogenously dissolved. In addition, a fraction δ of matches are exogenously destroyed every period.

2.2 Separations and values

The stock of unemployed after endogenous and exogenous separations into unemployment is:

$$u_{+}(x, z) = \nu \mathbf{1}\{x = \underline{x}\} + (1-\nu)\left[\sum_{x_{-1}} \pi_{xu}(x_{-1}, x)\, u(x_{-1}, z_{-1}) \right. \tag{1}$$

$$\left. + \sum_{x_{-1}} \sum_{y_{-1}} \left(\mathbf{1}\{S(x, y, z) < 0\} + \delta \mathbf{1}\{S(x, y, z) \geq 0\}\right) \pi_{xe}(x_{-1}, x)\, \pi_{y}(y_{-1}, y)\, h(x_{-1}, y_{-1}, z_{-1}) \right].$$

The stock of matches of type (x, y) at this point is:

$$h_{+}(x, y, z) = \sum_{x_{-1}} \sum_{y_{-1}} (1-\delta)(1-\nu)\, \mathbf{1}\{S(x, y, z) \geq 0\}\, \pi_{xe}(x_{-1}, x)\, \pi_{y}(y_{-1}, y)\, h(x_{-1}, y_{-1}, z_{-1}). \tag{2}$$

A worker that is unemployed during the production phase receives a flow payoff of $b(x, z)$ representing utility of leisure and value of home production. Due to the negotiation setup where the firm reaps the entire surplus above the worker's outside option, the value of unemployment, $B(x,z)$, is independent of the job finding probability, and any other endogenous variable or distribution:

$$B(x,z) = b(x,z) + \frac{1-\nu}{1+r} \sum_{x' \in X} \sum_{z' \in Z} B(x',z') \, \pi_{xu}(x,x') \, \pi(z,z') \quad . \tag{3}$$

where X is the set of human capital states and Z is the set of a aggregate productivity states.[10] As shown by LR (their proposition 1 and the proof thereof), the total surplus of a match does not depend on any of the (future) endogenous distributions. It is instead simply:

$$\begin{aligned} S(x,y,z) &= p(x,y,z) - b(x) \\ &+ \frac{(1-\delta)(1-\nu)}{1+r} \sum_{x' \in X} \sum_{y' \in Y} \sum_{z' \in Z} \max\left\{S(x',y',z'),0\right\} \pi_{xe}(x,x') \, \pi_y(y,y') \, \pi(z,z') \end{aligned} \tag{4}$$

where Y is the set of match-specific productivity states.

A measure one of new firms is created every period and their match-specific productivities are drawn from the probability density function (pdf) $f(y)$, which is identical across firms. Also, an unemployed worker exerts search effort s_0 and an employed worker exerts search effort s_1. Recalling that workers receive a value corresponding to their outside option, with the firm capturing the remaining part of the surplus, the expected value of a new match for a firm is:

$$\begin{aligned} J(z,\Gamma) &= \sum_{x \in X} \sum_{y \in Y} \frac{s_0}{L} u_+(x,z) \max\{S(x,y,z),0\} f(y) \\ &+ \sum_{x \in X} \sum_{y \in Y} \sum_{\tilde{y} \in Y} \frac{s_1}{L} h_+(x,\tilde{y},z) \max\{S(x,y,z) - S(x,\tilde{y},z),0\} f(y) \end{aligned} \tag{5}$$

where $\Gamma \equiv (h_+, u_+)$ and L is the aggregate amount of search effort:

$$L \equiv s_0 \sum_{x \in X} u_+(x,z) + s_1 \sum_{x \in X} \sum_{y \in Y} h_+(x,y,z) \, . \tag{6}$$

The first term in (5) refers to expected surplus from recruiting out of the pool of unemployed and the second term refers to expected surplus from recruiting from existing matches.

[10] In our calibration we assume that the flow income from unemployment is independent of the aggregate state, $b(x,z) = b(x)$. Given (3) this also implies $B(x,z) = B(x)$

2.3 Vacancy determination

After separations into unemployment, firms post vacancies and workers search. If a firm posts v vacancies it incurs a convex cost $c(v)$. The match-specific productivity is observed when the firm meets a worker, after the vacancy posting decision. The optimality condition for vacancy creation therefore implies:

$$c'(v) = qJ(z, \Gamma).$$

The vacancy cost function has the following functional form:

$$c(v) = \frac{c_0 v^{1+c_1}}{1+c_1}.$$

Thus, our model allows for convex vacancy posting costs which may affect the relationship between aggregate volatility and employment. As is shown in Table **??**, the effect of eliminating this convexity on employment is small.

Since all firms face the same probability distribution over productivities, aggregation in terms of vacancy posting across firms is trivially symmetric, i.e., $V = v$.

We assume the following Cobb-Douglas meeting function:

$$M \equiv \min\left\{\alpha L^{\omega} V^{1-\omega}, L, V\right\}. \tag{7}$$

Note that the probability of filling a vacancy (assuming an interior solution) is:

$$q = \frac{M}{V} = \alpha\theta^{-\omega}$$

where $\theta \equiv \frac{V}{L}$ is labor market tightness. Together with the matching function (7) and an assumption of an interior solution this implies that equilibrium vacancy postings are determined by:

$$v = V = \left(\frac{\alpha J(z, \Gamma)}{c_0 \theta^{\omega}}\right)^{1/c_1}. \tag{8}$$

By using (8), the definition of θ, and the definition of J in (5), labor market tightness is a function of z and Γ and can be written as:

$$\theta(z, \Gamma) = \left[\frac{1}{L}\left(\frac{\alpha J(z, \Gamma)}{c_0}\right)^{1/c_1}\right]^{\frac{c_1}{c_1+\omega}}. \tag{9}$$

2.4 Distributional dynamics

For a new match to be formed two conditions are required: the two parties must meet according to the meeting function (7) and the match must be an improvement over status quo (the current match or unemployment). The unemployment distribution $u(x,z)$ resulting from vacancy postings and search accordingly is:

$$u(x,z) = u_+(x,z)\left(1 - s_0\frac{M}{L}\sum_y \mathbf{1}\{S(x,y,z)\geq 0\}f(y)\right). \tag{10}$$

The corresponding expression for the employment distribution $h(x,y,z)$ is:

$$\begin{aligned} h(x,y,z) \quad = \quad & h_+(x,y,z) + \underbrace{u_+(x,z)\,s_0\frac{M}{L}\mathbf{1}\{S(x,y,z)\geq 0\}f(y)}_{\text{mass hired from unemployment}} \\ & \underbrace{-h_+(x,y,z)\,s_1\frac{M}{L}\sum_{\tilde{y}}\mathbf{1}\{S(x,\tilde{y},z) > S(x,y,z)\}f(\tilde{y})}_{\text{mass lost to more productive matches}} \\ & \underbrace{+s_1\frac{M}{L}\sum_{\tilde{y}} h_+(x,\tilde{y},z)\,\mathbf{1}\{S(x,y,z) > S(x,\tilde{y},z)\}f(y)}_{\text{mass poached from less productive matches}} \end{aligned} \tag{11}$$

where $\tilde{y}$ denotes the competing match when an employed worker is matched to a new job due to on-the-job search.

2.5 Wage determination

Let $W(w,x,y,z;\Gamma)$ denote the present value to a worker with human capital x, in a match with productivity y with wage w and aggregate productivity z, with Γ summarizing the endogenous aggregate state. These worker values are determined according to the sequential auction bargaining protocol in LR, and detailed as follows. Denote the renegotiated wage by w'. Workers hired out of unemployment receive their reservation wage w' such that

$$W\left(w',x,y,z;\Gamma\right) = B(x,z).$$

For employed workers that have received a poaching offer, Bertrand competition between employers imply that these workers have a present value $W(w,x,y,z;\Gamma)$ equal to the total surplus of the second best match that they have encountered during a spell of continuous employment. Formally, if a worker of type x, employed at a firm of type y meets a firm of type $\tilde{y}$ then, if $S(x,y,z) < S(x,\tilde{y},z)$ the worker switches to the new firm and gets the wage w' satisfying

$$W\left(w', x, \tilde{y}, z; \Gamma\right) = S\left(x, y, z\right) + B\left(x, z\right). \tag{12}$$

If instead $S\left(x, y, z\right) \geq S\left(x, \tilde{y}, z\right)$ the worker remains in his current match and gets a wage w' commensurate with the maximum of the value of the outside match $(x, \tilde{y})$ and the value at the current wage:

$$W\left(w', x, y, z; \Gamma\right) = \max\left\{S\left(x, \tilde{y}, z\right) + B\left(x, z\right), W\left(w, x, y, z; \Gamma\right)\right\}, \tag{13}$$

Wages for workers that do not receive poaching offers can also be rebargained, as aggregate or idiosyncratic shocks might affect whether the current wage is in the bargaining set, i.e.,

$$B\left(x, z\right) \leqslant W\left(w, x, y, z; \Gamma\right) \leqslant S\left(x, y, z\right) + B\left(x, z\right). \tag{14}$$

Along the lines of Hall (2005), the wage w is thus fixed within a match as long as it is in the bargaining set (14). In case the wage is too low or too high, violating (14), it is adjusted to generate a worker value at the closest boundary of the bargaining set.

Given the above protocol for how the worker value W is set, we are now ready to state an expression for the worker value function as a function of the current wage w. This expression includes the probability of an employed worker meeting a firm. Imposing an interior solution for M, $M = \alpha L^{\omega} V^{1-\omega}$ and using the definition of q, the probability of meeting a new firm is $s_1 \alpha \theta\left(z', \Gamma'\right)^{1-\omega}$. Then, given the wage w, the worker value is:

$$\begin{aligned} W\left(w, x, y, z; \Gamma\right) = w + \frac{1-\nu}{1+r} \sum_{x'} \sum_{y'} \sum_{z'} \pi_{xe}\left(x, x'\right) \pi_y\left(y, y'\right) \pi\left(z, z'\right) \\ \times [s' B\left(x', z'\right) + \left(1 - s'\right) \{\left(1 - s_1 \alpha \theta\left(z', \Gamma'\right)^{1-\omega}\right) W'_{np} \\ + s_1 \alpha \theta\left(z', \Gamma'\right)^{1-\omega} \sum_{\tilde{y}} \left(p^o_{\tilde{y}>y'} W'_{p,\tilde{y}>y'} + \left(1 - p^o_{\tilde{y}>y'}\right) W'_{p,\tilde{y}\leq y'}\right) f\left(\tilde{y}\right)\}] \end{aligned} \tag{15}$$

where

$$\begin{aligned} s' &= \left(\mathbf{1}\left\{S\left(x', y', z'\right) < 0\right\} + \delta \mathbf{1}\left\{S\left(x', y', z'\right) \geq 0\right\}\right) \\ W'_{np} &= \min\left\{S\left(x', y', z'\right) + B\left(x', z'\right), \max\left\{W\left(w, x', y', z'; \Gamma'\right), B\left(x', z'\right)\right\}\right\} \\ p^o_{\tilde{y}>y'} &= \mathbf{1}\left\{S\left(x', \tilde{y}, z'\right) > S\left(x', y', z'\right)\right\} \\ W'_{p,\tilde{y}>y'} &= S\left(x', y', z'\right) + B\left(x', z'\right) \\ W'_{p,\tilde{y}\leq y'} &= \max\left\{S\left(x', \tilde{y}, z'\right) + B\left(x', z'\right), W\left(w, x', y', z'; \Gamma'\right)\right\} \end{aligned}$$

where s' denotes separations, W'_{np} the value when not receiving a poaching offer, $p^o_{\tilde{y}>y',x',z'}$ a successful poaching offer, $W'_{p,\tilde{y}>y'}$ the value of a successful poaching offer and $W'_{p,\tilde{y}\leq y'}$ the value of an unsuccessful poaching offer. In expression (15), there are three possible cases for a worker that remains employed in the next period: i) not meeting any new firm, ii) receiving a successful poaching offer and switching jobs, and iii) receiving an unsuccessful poaching offer and staying in the old job.

2.6 Wage distribution

When determining the wage distribution, the current wage of the worker is a state variable. It summarizes the entire wage-relevant history of the worker. Conditional on aggregate technology evolving from z_{-1} to z, the distribution of matches over w, x and y evolves according to:

$$h^w_+ (w, x, y, z) = \sum_{x_{-1}} \sum_{y_{-1}} (1-\delta)(1-\nu)\,\mathbf{1}\{S(x,y,z)\geq 0\}\,\pi_{xe}(x_{-1},x)\,\pi_y(y_{-1},y)\,h^w(w,x_{-1},y_{-1},z_{-1})\,. \tag{16}$$

due to separations and idiosyncratic shocks. Analogously to (11) in section 2.4, we define $h^w(w,x,y,z)$ which accounts for new matches; see Appendix A.1.

3 Calibration

Flow payoff from unemployment is $b(x,z) = b_0 + b_1 x$, i.e. independent of the aggregate level of technology z. We let b depend on the human capital (i.e. productivity) of the worker, x, as a proxy for the value of home production or alternatively as a proxy for unemployment benefits that is a function of the previous earnings.

The log of the exogenous part of TFP, z, follows an AR(1) process approximated by a Markov chain. The log of the initial match productivity $f(y)$ is normally distributed. The log of the match-productivity y within a match follows an AR(1) process without drift that is approximated by a Markov chain. The mean value of this AR(1) process coincides with the mean value of the initial match productivity distribution, $f(y)$ and is normalized to 0.5.

The number of gridpoints for x, y and z are 10, 8 and 5 respectively.[11] The wage grid contains 15 points and is chosen separately for each parameter vector so as to only cover the relevant wage interval.[12] In constructing the grid for human capital, x, we, as Jarosch (2015) and Jung and Kuester

[11] Regarding y and z, we use Tauchen and Hussey's (1991) discretization of AR(1) processes with optimal weights from Flodén (2008). This algorithm has been shown by Flodén (2008) to be accurate also for processes with high persistence.

[12] The coarseness of the wage grid is less restrictive than it seems, as we map each wage to the two nearest grid points using the inverse of the distance to the grid point as weight.

(2011), follow Ljungqvist and Sargent (1998, 2008) in using an equal-spaced grid and in setting the ratio between the maximum and minimum value of x to 2. The structure of the transition matrices $\pi_{xe}(x, x')$ and $\pi_{xu}(x, x')$ for human capital also closely follow Ljungqvist and Sargent. Abstracting from the bounds, the probability of an employed worker to increase his human capital by one gridpoint is x_{up} and the probability for an unemployed worker to decrease his human capital by one gridpoint is x_{dn}. With the reciprocal probabilities the human capital of a worker is unchanged.

3.1 Calibration approach

The frequency of the model is monthly. We calibrate the model based on U.S. data in the following way: Parameters whose values are well established in the literature or from solid empirical evidence are set outside the model. Table 1 document these parameter values and their sources.

Table 1: Parameters set outside model

	Explanation	Value	Source
ω	Matching function elasticity	0.5	Pissarides (2009)
δ	Exogenous match separation rate	0.018	JOLTS and Fujita-Ramey
ν	Retirement rate	$1/(40 * 12)$	40 year work life
ρ	TFP shock persistence	0.960	Hagedorn-Manovskii
r	Interest rate	0.0041	Annual $r = 5\%$

The matching function elasticity ω is set in line with the convention in the literature. The exogenous match separation rate δ is equal to the ratio of non-layoffs in JOLTS 2001-2011 (0.598) multiplied by the mean E2U transition rate reported by Fujita and Ramey (2009), adjusted for workers finding a new job the same month as they lost the old job.[13] We set the retirement (or death) rate to match an average work life of 40 years, as e.g. Huckfeldt (2016). To compute the persistence of the AR process for TFP, we follow along the lines of Hagedorn and Manovskii (2008). Specifically, we simulate a monthly Markov chain to match a quarterly autocorrelation of (HP-filtered) log labor productivity of 0.765. Finally, we set r to yield an annualized interest rate of 5% as in LR.

The remaining parameters of our model do not have well established values and will instead be calibrated jointly to match key moments. Note first that, since we are interested in the cost of business cycles, it is important to match GDP and unemployment volatility. We calibrate the 10 parameters in Table 3 by matching the 10 moments in Table 2 by minimizing the squared percentage deviation between model and data moments. The model parameters are jointly estimated, but some moments are more informative about certain parameters. The transition rates from unemployment to employ-

[13] The latter implies that the separation rate exceeds the E2U rate by a factor 1/(1-job finding rate). By using Fujita and Ramey's number for E2U transitions we control for the fact that empirically, but not in our model, workers flow in and out of the labor force.

ment tend to be informative about matching function productivity α and the vacancy cost parameter c_0. The job to job transition rate is informative about the relative search intensity of employed s_1/s_0, the volatility of GDP and unemployment about the standard deviation of the aggregate productivity process. Moreover, the correlation between separations and labor productivity and earnings loss cyclicality are informative about the match-specific shock process parameters.[14] Wage dispersion and the earnings loss level (and cyclicality) are informative about the human capital gain/loss when employed/unemployed, x_{up} and x_{dn}.[15] Wage dispersion and unemployment volatility are informative about the unemployment payoff parameters. Finally, GDP persistence is informative about parameters generating endogenous persistence in the model. These include any parameters directly affecting labor market flows as well as the human capital dynamics.

Table 2: Moments to Match in Calibration of the Model

Moment	Data source	Target value (data)	Model value
U2E transition rate, mean	Fujita-Ramey (2009)	0.340	0.255
J2J transition rate, mean	Moscarini-Thompson	0.0320	0.0325
E2U transition rate, mean	Fujita-Ramey (2009)	0.020	0.0218
Corr(E2U trans rate,labor prod)	Fujita-Ramey (2012)	-0.52	-0.523
Unemployment, std.dev.	BLS 1980-2010	0.107	0.117
GDP, std.dev.	BEA 1980-2010	0.0136	0.0140
GDP, persistence	BEA 1980-2010	0.867	0.839
Earnings loss level	DvW	22.4%	16.0%
Earnings loss cyclicality	DvW	40.1%	38.1%
Wage disp: Mean-min ratio	Hornstein *et al*	1.50	1.39

Notes: U2E, J2J and E2U transition rates are at a monthly frequency. Labor-market variables used for computing correlations and standard deviations (E2U transition rate, labor productivity and unemployment) are quarterly means of monthly series. These and GDP and have been logged and HP-filtered with $\lambda = 1600$, both in the data and the model. Earnings loss is measured on an annual frequency.

The relevant measure of wage dispersion for our model is "residual" wage dispersion, i.e. controlling for heterogeneity not present in the model such as age, education, etc. We take the mean-min ratio (capturing the minimum by the 10th wage percentile) from Hornstein, Krusell and Violante (2007) as

[14] We keep the variance of the initial match-specific productivity, $f(y)$, equal to the ergodic variance of the AR(1) process for the within-match productivity dynamics.

[15] As in Jarosch (2015), we impose a relationship between x_{up} and x_{dn} such that the number of increases in human capital roughly equals the number of decreases to minimize bunching at end-points of the human capital grid X. In particular, let $u^e(x)$ (and $h^e(x, y)$) denote the ergodic distribution of unemployed (employed) workers across x, given that aggregate productivity is constant (and set to one) for all t. Then, letting $u^{tot} = \sum_x u^e(x)$, we impose $(1-\nu)\, x_{up}\left(1-u^{tot}\right)\Delta x = (1-\nu)\, x_{dn} u^{tot}\Delta x + \nu\left(E_x\left(\sum_y h^e(x, y) + u^e(x)\right) - \underline{x}\right)$ for the equilibrium values of employment and unemployment, implying $x_{dn} = \left(x_{up} - \frac{\nu}{1-\nu}\frac{\left[E_x\left(\sum_y h^e(x,y)+u^e(x)\right)-\underline{x}\right]}{(1-u^{tot})\Delta x}\right)\frac{1-u^{tot}}{u^{tot}}$. There will still be some upward drift, and thereby upper end-point bunching, in the human capital distribution if an above proportional fraction of the unemployed are at the lower bound of the human capital grid, unless this is offset by the analogous force of above proportional fraction of employed workers at the upper bound.

our measure of wage dispersion. We use their preferred measure of 1.50 which is an average of census, OES and PSID data.

We use the same definition of earnings loss from job displacement as the empirical literature. We target the average level of earnings loss as a fraction of pre-displacement earnings in the second full calendar year (which ends 25-35 months after displacement) reported by DvW. The second year is chosen as a trade-off between two factors: First, the aim of this moment is to capture human capital loss during unemployment, and human capital loss is the main factor driving earnings loss from job displacement at long horizons. This speaks in favor of targeting earnings loss moments at long horizons, even longer than 2-3 years. Second, computational considerations favors shorter horizons.[16] This also favors using earnings losses instead of wage increases during employment to pin down human capital dynamics, since most of the human capital loss is materialized relatively quickly, due to fairly short unemployment durations. We also target the cyclical component of earnings loss, measured as the percentage increase in earnings loss in recessions compared to expansions, again measured for the second full calendar year after displacement.

3.1.1 Earnings loss computations

To compute earnings loss cyclicality, we first need to define what constitutes an expansion and a recession. In the model we define a time period, i.e. a month, as an expansion (recession) if GDP is above (below) the 12th percentile in the simulated output of our model. The choice of the 12th percentile as the cutoff between expansion and recession follows Huckfeldt (2016) and is made to enable comparison with DvW. They use NBER dated recessions and these make up 12% of their sample period.

The observation frequency of earnings in the empirical literature is annual. We accordingly compute earnings losses of workers displaced in a particular calender year. When computing earnings losses we follow DvW by weighting each expansion (or recession) year by the number of months of that year that the economy was in an expansion (recession). For workers displaced in a given expansion (recession) month, we then compute average earnings for displaced (defined as separated, either endogenously or exogenously) and non-displaced workers, respectively, for each period over the earnings loss horizon. We let the "control group" of non-displaced workers be identical to the displaced workers in terms of all individual state variables, w, x, y, in the month prior to displacement. In this

[16] Earnings loss computations make up the vast majority of the computational time for our model.

way we minimize the selection effects in generating earnings losses.[17,18]

4 Results

4.1 Targeted moments and the parameter estimates

The moment-matching exercise can be evaluated by comparing the last two columns in Table 2. The model is able to fit most of these moments well. The two exceptions are the level of earnings loss and the U2E transition rates, with deviations of roughly 25%.

The above moment matching exercise determines the 10 parameters in Table 3. It might appear surprising that we need to calibrate the volatility of (the exogenous part of) TFP, but this is necessary as the model has internal amplification of the exogenous TFP shocks, as both the productivity of matches, the level of human capital and sorting between workers and jobs varies over the cycle. All of this implies that TFP in our model is a combination of exogenous TFP and endogenous propagation.

Table 3: Parameters obtained by moment-matching

Parameter	Explanation	Value
α	Matching function productivity	0.527
s_1/s_0	Relative search intensity of employed	0.350
c_0	Vacancy cost level	0.306
c_1	Vacancy cost curvature	0.254
x_{up}	Human capital loss, probability	0.068
b_0	Unemployment payoff intercept	0.046
b_1	Unemployment payoff coeff. x	0.855
ρ_y	Match-specific shock persistence	0.573
σ_y	Match-specific shock std.dev.	0.187
$100\sigma_z$	TFP shock std.dev.	0.469

Let us here interpret and comment on some of the parameter values in Table 3 obtained through the moment matching exercise. The value for s_1/s_0 indicates that employed workers meet prospective employers roughly one-third as often as unemployed workers. The value of c_1 of one quarter indicates that vacancy posting costs are close to linear, only slightly convex. We follow LR and report the replacement ratio for unemployed workers as a fraction of the output of the best possible match. b_0 and b_1 jointly imply that this ratio is 0.90, averaged over the human capital values. The estimate of the volatility of the exogenous part of TFP (to be exact, labor productivity) imply that roughly half

[17] Note that, given that we know how to compute transitions using the equations for $u_+(x_{-1}, z_{-1})$, $u(x,z)$, $h^w_+(w, x_{-1}, y_{-1}, z_{-1})$ and $h^w(w,x,y,z)$, we can feed in an arbitrary initial worker distribution and follow the distribution of these workers over time. Then, we can use the resulting distributions to compute earnings in the two groups.

[18] As in the empirical literature, e.g. DvW, we require that non-displaced workers stay with the same employer for the first 3 years after the displacement date. This requirement slightly modifies the expressions for $h^w_+(w, x_{-1}, y_{-1}, z_{-1})$ and $h^w(w,x,y,z)$.

of the total variation in observed labor productivity is exogenous. The other half is due to cyclical variation in endogenous variables. That the level of cyclical amplification implied by the two-sided heterogeneity in the labor market is this strong, is an interesting result in it own right.

Given their centrality for our mechanism, we report and comment in more detail on our estimates of the learning on-the-job parameters. The estimated Markov transition probabilities ($x_{up} = 0.068$ and $x_{dn} = 0.76$) imply that the expected monthly human capital increase for an employed worker is 0.35 percent, while the expected decrease when unemployed is 2.54 percent.[19]

We know of only one direct measure in the literature of general human capital loss while non-employed: Edin and Gustavsson (2008). They use a Swedish panel of individual level data that includes test results on labor market relevant general skills and information about employment status between test dates. First, they find that time-out-of-work (compared to employment) implies skill loss, significant at the 1% level. Second, this skill loss appears to be linear in time out-of-work. Third, the speed of skill loss is substantial: being out-of-work for a year implies losing skills equivalent to 0.7 years of schooling.[20]

The human capital dynamics can be compared to estimates in models broadly similar to ours. Huckfeldt (2016) reports a 0.33 percent expected monthly human capital increase for workers in skill-intensive jobs (0.22 percent in skill-neutral jobs).[21] Jarosch (2015) reports only the monthly human capital Markov transitions probabilities: 0.0141 for employed and 0.131 for unemployed. For the employed worker with the mid-point of human capital this implies an expected increase of 0.13 percent and for the unemployed worker with the mid-point of human capital it implies a 1.2 percent decrease. To sum up this comparison to the literature, our human capital dynamics are roughly in line with the estimates of Huckfeldt (2016), but the speed of learning and "unlearning" is substantially above what Jarosch (2015) finds using German data.

4.2 Welfare measure

Our main exercise is to look at the welfare consequences of eliminating aggregate volatility, implying that we take the transition dynamics into account. As is standard in the literature, we report the

[19] These values takes into account the distribution of employed and unemployed workers across the human capital grid, including the effects of the bounds of the human capital grid.

[20] There is an older empirical literature that attributes all wage loss when re-employed after an unemployment spell to human capital loss, see e.g. Keane and Wolpin (1997). This abstracts from selection effects in the unemployment pool and the potential tendency to accept worse jobs the longer the unemployment spell. In addition, this is not consistent with our model so we abstract from that literature.

[21] The comparison of losses when unemployed to Huckfeldt's results is clouded by the fact, differently from our model, he allows for both gradual and sudden loss of human capital during unemployment. Our human capital loss estimates for unemployed workers will therefore tend to be higher than his, which is 1.1 percent.

amount of consumption agents are willing to forego to eliminate business cycles. The linearity of utility in consumption makes welfare calculations straight forward in our model, as per period aggregate welfare is equal to aggregate consumption (i.e. GDP net of vacancy posting costs). Note that one may interpret the unemployment payoff, b, in two ways which have different welfare implications. In the first interpretation, b is home production (or equivalently, from a welfare perspective, utility of leisure) in which case the welfare relevant quantity is the sum of market consumption and the unemployment payoff. In the second interpretation b is a pecuniary transfer not directly affecting aggregate utility. We report results for both interpretations.[22]

To further facilitate the interpretation of the results, we also report the results for a comparison between the stochastic average and the non-stochastic steady state.

4.3 Results for costs of business cycles

As outlined above we compute the welfare effects of eliminating aggregate volatility from the economy. As summarized in Table 4 we find that in our model the elimination of aggregate shocks, taking the transition into account, increases welfare by 2.57% or 1.52%, depending on the interpretation of unemployment benefits.

Table 4: Welfare effects of eliminating business cycles)

Welfare, b transfer	2.57
Welfare, b home prod	1.52

We can gain some intution into the mechanism by instead comparing the steady state quantities in an economy with and without aggregate volatility. As summarized in Table 5 we find that in our model the elimination of aggregate volatility increase employment, GDP and consumption by substantial amounts. In particular, consumption, and thereby (steady state) welfare, increase by 2.7-4.1 percent, depending on the interpretation of unemployment benefits. We note that the key force generating cost of business cycles is the human capital dynamics. The mean level of human capital of employed workers, $E\left(x \times h\left(\cdot\right)\right)$, increase by 2.2 percent when aggregate volatility is eliminated. The corresponding number for unemployed workers, $E\left(x \times u\left(\cdot\right)\right)$, is 4.5 percent.[23] Both of these changes are due to the increase in employment and the corresponding decrease in unemployment. Note that there are other factors affecting GDP than just the mean values of employment and human capital. Examples include the change in the mean level of match-specific productivity, $E\left(y \times h\left(\cdot\right)\right)$, (which

[22] In case b consists of both home production and transfers, our results below gives upper and lower bounds for the welfare cost.

[23] Although the human capital of unemployed workers has no direct effect on output, it is still relevant as it is an important determinant for firms' incentives to post vacancies.

decreases by 0.16 percent) and the changed degree of sorting between workers and firms (as well as the covariation between any of these objects with the cycle).

Table 5: Gains from removing business cycles (in percent)

GDP	4.13
C=GDP-vacancy cost	4.13
C=GDP-vacancy costs+$b * u$	2.65
Employment	2.02
$E\left(x \times h\left(\cdot\right)\right)$	2.20
$E\left(x \times u\left(\cdot\right)\right)$	4.52
$E\left(y \times h\left(\cdot\right)\right)$	-0.16

5 Conclusions

A central question in macroeconomics is how large the welfare costs of business cycles are. We show that cyclical variation reduces aggregate welfare in a labor market search model with general human capital dynamics, as it drives down the level of employment, output and consumption. The key mechanism of the paper is the following: Empirically, the Beveridge correlation is negative, i.e. vacancies and unemployment are negatively correlated. This, in turn, means that business cycles tend to reduce the average number of matches and hence employment, through the employment-flow equation. Then, since learning on-the-job imply that human capital is increasing in the employment rate, it follows that aggregate volatility reduces human capital. This, in turn, reduces incentives to post vacancies, further reducing employment. We find that the steady state output and welfare losses due to business cycles are large - they amount to 4.1 percent and 2.7-4.1 percent respectively, and this result is not very sensitive to the exact calibration. Accounting for the transition dynamics the welfare gains of eliminating business cycles are slightly smaller, 1.5-2.6 percent.

To conclude, let us briefly discuss some broader implications of our results. In our model there is only one type of aggregate shock. If we view this shock as a “catch-all” for any variation in firm revenues including effects of fiscal and monetary policy we can draw interesting policy conclusions. In particular, a policy that successfully stabilizes unemployment (or job finding rates) raises the average level of output. For this reason, our paper rationalizes an unemployment stabilization mandate for central banks as well as a fiscal policy that stabilizes unemployment. In this sense we reach the same conclusion as Berger et al. (2016) and Galí (2016) but for a very different reason. Berger et al.’s argument is about unemployment stabilization reducing idiosyncratic risk related to layoffs while Galí’s mechanism is about hysteresis due to insider-outsider dynamics. Our argument is about unemployment stabilization leading to a higher average level of output, thereby more closely related to the statement by Summers (2015) that stabilization policy can have major effects on average levels of output over periods of decades.

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A Appendix

A.1 Employment transitions

When accounting for the wage distribution, the employment transition follows:

$$
\begin{aligned}
& h^w\left(w^*, x, y, z\right)= \\
& h_{+}^w\left(w^*, x, y, z\right)-\underbrace{h_{+}^w\left(w^*, x, y, z\right) s_1 \frac{M}{L} \sum_{\tilde{y}} p_{\tilde{y}>y}^o f(\tilde{y})}_{\text{mass lost to more productive matches}} \\
& \underbrace{-h_{+}^w\left(w^*, x, y, z\right) s_1 \frac{M}{L} \sum_{\tilde{y}} \mathbf{1}\left\{S(x, \tilde{y}, z)+B(x, z)>W\left(w^*, x, y, z ; \Gamma\right)\right\}\left(1-p_{\tilde{y}>y}^o\right) f(\tilde{y})}_{\text{mass lost to higher wage offers from less productive matches}} \\
& \underbrace{+s_1 \frac{M}{L} \sum_{\tilde{y}} \sum_{\tilde{w}} h_{+}^w(\tilde{w}, x, y, z) \mathbf{1}\left\{w(\tilde{w}, x, y, z ; \Gamma)=w^*\right\}\left(1-p_{\tilde{y}>y}^o\right) f(\tilde{y})}_{\text{mass gained from increased wage due to offers from less productive matches}} \\
& \underbrace{+s_1 \frac{M}{L} f(y) \sum_{\tilde{y}} h_{+}(x, \tilde{y}) \mathbf{1}\left\{W\left(w^*, x, y, z ; \Gamma\right)=S(x, \tilde{y}, z)+B(x, z)\right\} p_{y>\tilde{y}}^o}_{\text{mass poached from less productive matches}} \\
& \underbrace{-h_{+}^w\left(w^*, x, y, z\right) \mathbf{1}\left\{W\left(w^*, x, y, z ; \Gamma\right) \notin B S(x, y, z)\right\}}_{\text{mass lost due to being outside bargaining set}} \\
& \underbrace{+\sum_{\tilde{w}} h_{+}^w(\tilde{w}, x, y, z) \mathbf{1}\left\{w(\tilde{w}, x, y, z ; \Gamma)=w^*\right\} \mathbf{1}\{W(\tilde{w}, x, y, z ; \Gamma) \notin B S(x, y, z)\}}_{\text{mass gained from other wages being outside bargaining set}} \\
& \underbrace{+s_0 \frac{M}{L} u_{+}(x) f(y) S_{x y z} \mathbf{1}\left\{W\left(w^*, x, y, z ; \Gamma\right)=B(x, z)\right\}}_{\text{mass hired from unemployment}}
\end{aligned} \tag{17}
$$

where

$$
\begin{aligned}
B S(x, y, z) & = [B(x, z), S(x, y, z)+B(x, z)] \\
S_{x y z} & \equiv \mathbf{1}\{S(x, y, z) \geq 0\} \\
p_{y>\tilde{y}}^o & \equiv \mathbf{1}\{S(x, y, z)>S(x, \tilde{y}, z)\} \\
p_{\tilde{y}>y}^o & \equiv \mathbf{1}\{S(x, \tilde{y}, z)>S(x, y, z)\}
\end{aligned}
$$

A.2 Solution algorithm

Step 1. Obtain the equilibrium without aggregate volatility by the following substeps:

i) Use value function iteration to solve for $S(x,y,\bar{z})$ and $S(x,y,z)$ in (4)

ii) Make an initial guess for equilibrium unemployment.

iii) Set the parameter $x_{dn} = \left(x_{up} - \frac{\nu}{1-\nu} \frac{[E_x(h(x)+u(x))-\underline{x}]}{(1-u^{tot})\Delta x} \right) \frac{1-u^{tot}}{u^{tot}}$ where $u^{tot} = \sum_x u^e(x)$ with $u^e(x)$ denoting the ergodic distribution of unemployed workers across x, given that aggregate productivity is set to one for all t.

iv) Use value function iteration to solve for $B(x, \bar{z})$ in (3).

v) Compute the ergodic distributions for $u(x)$ and $h(x, y)$ for a fixed $z = \bar{z}$ (see below for details).

vi) If unemployment is too different from previous value, go back to iii).

Step 2. Use value function iteration to solve $B(x, z)$ in (3).

Step 3. Solve for $\{J_t, h_{t+1}, u_{t+1}, V_t, L_t, M_t, \Gamma_t\}_{t=0}^{T}$ recursively for each time period. Given the solution for $S(x, y, z)$, the initial conditions u_0 and h_0, and a sequence for $\{z_t\}_{t=0...T}$, iterate forward to create a time series for u_t, h_t and any aggregates of these we are interested in:

i) calculate $u_{t+}(x)$ and $h_{t+}(x, y)$ using (1) and (2)

ii) calculate L_t by aggregating over $u_{t+}(x)$ and $h_{t+}(x, y)$

iii) calculate J_t using (5).

iv) calculate θ_t using (9)

v) calculate V_t using (8)

vi) calculate $u_{t+1}(x)$ and $h_{t+1}(x, y)$ using (10) and employment transition (11)

To obtain the ergodic distributions for $u_{t+1}(x)$ and $h_{t+1}(x, y)$ simulate above for a fixed z until convergence in these distributions.

Given the sequence based on $\{z_t\}_{t=0...T}$ above, we use the resulting sequence of θ (after removing an initial burn-in period) to compute wages and then the sequence of h^w_{t+1} to compute relevant moments of the wage distribution along the sequence where we have followed the algorithm described in section A.2.1 to compute worker values $W(w, x, y, z; \Gamma)$ and wages $w(w, x, y, z; \Gamma)$.

A.2.1 Algorithm for determination of W and w

As can be seen from (15) the worker value function depends on Γ', i.e. the entire expected next period distribution of matches across x and y and unemployed workers distribution over x. The challenge is to reduce the dimensionality of the distributions Γ' to something manageable. The key to our algorithm is to note that all influence of the endogenous distributions goes through the next period labor market tightness, θ'. In addition, according to (9) labor market tightness is a function only of L in (6) and J in (5). Hence, we can write θ as a function of the moments that make up (6) and (5); $\theta = \Theta(m_1, m_2, m_3; z)$. In particular, based on (6) and up to a scalar transformation,

$$m_1 = \sum_x u_+(x, z)$$

as we note that $\sum_x \sum_y h_+ (x, y, z) = 1 - \sum_x u_+ (x, z)$ and accordingly $L_t \equiv s_0 \sum_x u_+ (x, z) + s_1 (1 - \sum_x u_+ (x, z))$. Given L_t from m_1, equation (5) implies that J is fully determined by the following two terms:

$$m_2 = \sum_x \sum_y u_+ (x, z) \max \{S (x, y, z), 0\} f (y)$$

and

$$m_3 = \sum_x \sum_y \sum_{\tilde{y}} h_+ (x, \tilde{y}, z) \max \{S (x, y, z) - S (x, \tilde{y}, z), 0\} f (y).$$

To compute next period values of these moments we assume a linear relationship to today's moments. Thus, we write

$$m'_m = H_m (m_1, m_2, m_3, z')$$

We loosely follow Krusell and Smith (1998). Since we can compute the evolution of the distributions u_+ and h_+ and θ without solving for wages and values, we generate a sequence of aggregate productivity shocks, compute m_i and tightness θ and, given a linear functional form of H_m and Θ, then estimate the functions H_m and Θ. Given the above arguments it is unsurprising that the R^2 of the function $\Theta (m_1, m_2, m_3)$ is approximately unity (> 0.995). It turns out that $H_m (m_1, m_2, m_3, z')$ also has a reasonably high R^2.

In the end, we can replace the distributions in Γ' by (m_1, m_2, m_3) so that instead of $(w, x, y, z; \Gamma)$ the final state vector is $(w, x, y, z; m_1, m_2, m_3)$. With the functions Θ and H_m at hand, we solve for worker values W. This is done with value function iteration. m_i, $\{i = 1, 2, 3\}$ is therefore discretized on a grid with 2 gridpoints. We choose fewer gridpoints for m_i than for z as m_i is quantitatively less important.

Finally, once we know the worker values W we can solve for wages w residually. This amounts to rewriting equation (15) to find the wage that yields the right value of W for the current state vector $(w, x, y, z; m_1, m_2, m_3)$ given the expected future values for the worker.