Business Cycle Asymmetries and the Labor Market*

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This paper shows that a search and matching model with idiosyncratic training cost shocks can explain the asymmetric movement of the job-finding rate over the business cycle and the decline of matching efficiency in recessions. Large negative aggregate shocks move the hiring cutoff into a part of the training cost distribution with higher density. The position of the hiring cutoff in the distribution is disciplined by the empirical elasticity of the job-finding rate with respect to market tightness. Our model explains a large fraction of the matching efficiency decline during the Great Recession and generates state-dependent effects of policy interventions.

JEL Classification: E24, E32, J63, J64

Key words: Business cycle asymmetries, matching function, Beverdige curve, job-finding rate, unemployment, effectiveness of policy

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1. Introduction

There is a long-established literature that documents that employment and unemployment in the United States behave asymmetrically over the business cycle, i.e. the downward movement of employment in recessions is much stronger than the upward movement in booms (e.g. Neftçi, 1984; McKay and Reis, 2008; Abbritti and Fahr, 2013). In recent years, the experience of the Great Recession gave rise to a separate literature documenting a decline of matching efficiency (e.g. Barnichon and Figura, 2015) and an outward shift of the Beveridge curve in recessions (Diamond and Şahin, 2015).

While both facts – unemployment asymmetry as well as shifts of the Beveridge Curve and matching efficiency – are well known to the literature, we are the first to show that these are two sides of the same coin. We document that an asymmetric job-finding rate is an important driver of unemployment asymmetry. The time series distribution of the job-finding rate is skewed to the left, which means that in severe recessions, the downward adjustment of the job-finding rate is particularly strong. By contrast, market tightness moves more symmetrically over the business cycle. In severe recessions, the decline of the job-finding rate is typically stronger than predicted by a standard Cobb-Douglas constant returns matching function that features a constant elasticity between the job-finding rate and market tightness (vacancies divided by unemployment). Through the lens of a standard matching function, this pattern of the data is interpreted as a decline of the matching efficiency.

We provide an economic mechanism explaining these facts by enhancing a standard search and matching model with a selection stage (Chugh and Merkl, 2016; Sedláček, 2014). In our model, matching is a two-stage process. First, workers and firms have to meet. We call this the contact stage, during which contacts form according to a standard Cobb-Douglas contact function. At a second stage, firm and worker assess whether they are a good fit and the firm decides whether to hire the worker. We call this the selection stage. We model the selection stage as a draw from a match-specific training cost distribution. This means that firms will only select a fraction of contacts for hiring. Over the cycle, firms endogenously adjust their hiring standards. In a recession, when profit outlooks are bleak and market tightness is slack, firms will increase their hiring standards and only select workers with relatively low training costs. The hiring threshold for training costs and hence the selection rate decrease. In a boom, when profit outlooks are favorable and the market is tight, firms will increase their hiring threshold and accept workers with higher match-specific training costs. Thus, the selection rate increases. We show that the response of the selection rate and hence the job-finding rate is stronger in recessions than in booms whenever the hiring cutoff is at a downward-sloping part of the training cost distribution. We prove that this condition is satisfied for empirically plausible values of the elasticity of the job-finding rate with respect to market tightness: in our model, this moment is closely tied to the curvature of the idiosyncratic training cost distribution at the hiring cutoff.[1] Furthermore, the nonlinearity inherent in the selection

[1]More precisely, we calibrate the elasticity of the job-finding rate in the simulated model to the one in the data. The underlying curvature of the idiosyncratic training cost distribution at the cutoff point is a key determinant for the estimated elasticity based on simulated data. Matching the elasticity

mechanism also implies that the elasticity of the job-finding rate with respect to market tightness is no longer constant in our model – as assumed with a standard matching function – but changes when the hiring cutoff moves over the cycle. Looking at the data through the lens of a constant elasticity matching function, this appears as a decline of matching efficiency. Thereby, our model can explain both phenomena from the data: (i) asymmetric reactions of the job-finding rate, employment and unemployment over the business cycle, (ii) the decline of the estimated matching efficiency and the outward shifts of the Beveridge curve in recessions. Quantitatively, the reaction of the job-finding rate in our model is substantially larger in response to a negative shock compared to a positive shock. For a realistic slump of the labor market, our model endogenously accounts for the majority of the decline of matching efficiency during the Great Recession. In addition, we show that the proposed mechanism is of major importance for understanding the quantitative effects of policy interventions in booms and in recessions. These depend very much on the business cycle state.

We believe that labor selection (i.e. endogenous hiring standards) is an economic mechanism that is of great relevance. Based on the cross-sectional Employment Opportunity Pilot Project (EOPP), Barron et al. (1985, p. 50) document for the United States that "(...) most employment is the outcome of an employer selecting from a pool of job applicants (...)." More recently, Faberman et al. (2017) show based on a supplement to the Survey of Consumer Expectations that only a fraction of worker-firm contacts translate to job offers. While to our knowledge there is no time-series evidence for the United States yet, Hochmuth et al. (2018) use the German IAB Job Vacancy Survey to construct aggregate time series (national, sector, state level) for the share of applicants that is selected over time (selection rate). They document that the selection rate moves procyclically over the business cycle and shows much stronger fluctuations than real GDP. Importantly, the selection rate accounts for about half of the movement of the job-finding rate over the business cycle. Kohlbrecher et al. (2016) fit an idiosyncratic productivity distribution to match the distribution of residual entry wages in Germany and find that selection drives about two thirds of the movement of the job-finding rate. Thus, Kohlbrecher et al. (2016) and Hochmuth et al. (2018) provide evidence for Germany that (in addition to the well-established contact channel) labor selection is an important margin over the business cycle. In line with our theoretical model, there is also direct evidence for the United States that training costs are large and heterogeneous across workers. Based on the EOPP, Barron et al. (1989) show that on average firms spend 150 hours for on-the-job training of newly hired workers.

Furthermore, the business cycle behavior of labor selection in our model is well in line with the empirical evidence on recruiting intensity. According to Davis et al. (2013) and Gavazza et al. (2018), a collapse of recruiting intensity can explain the decline of matching efficiency in the Great Recession. Increased hiring standards could be one important dimension of recruiting intensity, although there may be many others. While our paper focuses on the time series dimension of the data, Baydur (2017) shows that a selection model can also replicate important cross-sectional dimensions of the data (e.g.

from the data requires the cutoff point to be at a downward sloping part of the distribution.

the cross-sectional behavior of vacancy yields, as outlined by Davis et al. (2013)).

How do our results compare to the emerging literature on the ability of search and matching models to generate labor market asymmetries? Several recent papers show that the standard search and matching model can generate asymmetric responses of the job-finding rate and unemployment. Abbritti and Fahr (2013) propose a model with downward real wage rigidity, while Petrosky-Nadeau and Zhang (2017), Petrosky-Nadeau et al. (2018), and Ferraro and Fiori (2018) use calibrations in the spirit of Hagedorn and Manovskii (2008) to generate asymmetric effects.[2] We differ from these papers in several dimensions. First, none of these papers has a second stage of hiring. Thus, by definition the job-finding rate is a stable function of market tightness. By contrast, our approach not only explains the asymmetry of unemployment and the job-finding rate but also the observed decline of the matching efficiency in recessions. We are therefore the first to explain and relate both empirical phenomena. Second, the ability of a model to generate quantitatively meaningful asymmetric labor market responses is tightly connected to amplification. Given the well-known lack of amplification in the standard search and matching model (Shimer, 2005), the existing literature either relies on some form of wage rigidity (as in Hall, 2005) and/or small surplus calibration (as in Hagedorn and Manovskii, 2008). However, the cyclicality of real wages over the business cycle is a highly debated issue (e.g. Haefke et al., 2013; Gertler et al., 2016). Finally, we show that both in our model and in a search and matching model without labor selection the asymmetry of the job-finding rate is tightly connected to the elasticity of the job-finding rate with respect to market tightness. We show that for a given elasticity and a given amplification of the job-finding rate, the asymmetry generated by the model with labor selection is substantially larger (see Appendix E).

The focus of our paper is to explain the asymmetric nature of job-creation and its connection to the decline of matching efficiency in recessions. It is well known that the separation rate into unemployment (although not necessarily the total separation rate) is highly skewed over the business cycle (see e.g. Ferraro, 2018). This also contributes to the asymmetry of the unemployment rate. We show in the Appendix an extension of our model that includes endogenous separations and persistent idiosyncratic shocks. As expected, the separation margin further adds to the skewness of the unemployment rate. Importantly, our results regarding the job-creation margin and the matching efficiency are unaffected by this extension. The qualitative and even quantitative responses of the selection and job-finding rate are very similar and matching efficiency drops in recessions. A well known caveat of the endogenous separation model is the collapse of the Beveridge curve. Of course, this might be addressed with a more complex model. As this is not the focus of our paper, we prefer to keep our model simple and tractable.

One of the key insights of our paper is that labor selection can generate asymmetric responses of the job-finding rate and shifts of matching efficiency. Would other economic mechanisms generate similar results? One potential alternative explanation is endogenous search effort. If workers search less in recessions or the composition of the unemployment pool changes towards long-term unemployed with lower search effort (see

[2]For a discussion of labor market asymmetries in models with different submarkets see Section 7.

Hall and Schulhofer-Wohl, 2018), this could lead to a decline of estimated matching efficiency and asymmetries of the job-finding rate. However, Mukoyama et al. (2018) show that search effort is actually countercyclical in the US and Hornstein and Kudlyak (2016) show that this countercyclicality and compositional shifts of the unemployment pool cancel out during the Great Recession.

The rest of the paper is structured as follows. Section 2 presents stylized facts on business cycle asymmetries and the cyclicality of matching efficiency. Section 3 presents a search and matching model with labor selection. Section 4 provides analytical results. Section 5 outlays our calibration strategy and Section 6 presents numerical results in the fully nonlinear setting. Section 7 puts our results in perspective to the existing literature.

2. Stylized Facts

This section documents some stylized facts on business cycle and labor market asymmetries. We complement the existing literature with new insights, with particular emphasis on the nonlinear trajectory of the Beveridge curve and the matching function.

Stylized Fact 1. *The unemployment rate is positively skewed. It increases more in recessions than it decreases in booms.*

Figure 1 shows the annualized growth rates of output and unemployment for quarterly time series from 1951:I to 2016:IV for the United States (see Appendix A for a data description). The unemployment rate moves a lot more in recessions than in booms.[3] This asymmetry has been widely documented in the literature before (see e.g. Abbritti and Fahr, 2013; McKay and Reis, 2008).

Stylized Fact 2. *The job-finding rate is negatively skewed. It decreases more in recessions than it increases in booms.*

The skewness of unemployment may appear unsurprising because the standard unemployment equation is highly nonlinear.[4] However, Figure 2 illustrates that the job-finding rate is also strongly skewed, although in the opposite direction as unemployment. The job-finding rate falls a lot more in recessions than it increases in booms.[5] Thus, the strong decline of the job-finding rate in recessions is an important driver for the asymmetric increase of the unemployment rate. This is also documented by Ferraro (2018).

The separation rate based on the dataset by Shimer (2012) is also strongly skewed and contributes to the skewness of unemployment (see Ferraro, 2018). However, in the JOLTS dataset the spike of layoffs in the Great Recession was completely compensated

[3] This is both true for the unemployment level and the unemployment rate. The unemployment rate is also skewed when absolute instead of percent deviations are taken.

[4] Consider the steady state unemployment rate $u = \frac{sr}{jfr+sr}$, where sr is the separation rate and jfr is the job-finding rate. Even with a symmetric job-finding rate, unemployment would be skewed. This can be seen by taking the second derivative with respect to the job-finding rate: $\frac{\partial^2 u}{\partial jfr^2} > 0$.

[5] The skewness of the job-finding rate time series becomes even more pronounced when looking at absolute instead of percent deviations.

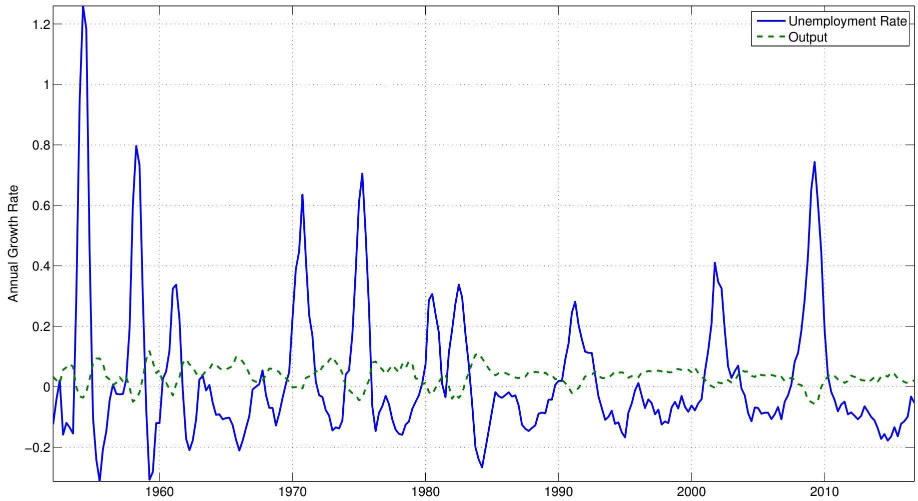

Figure 1: Annual growth rates of unemployment rate and real GDP.

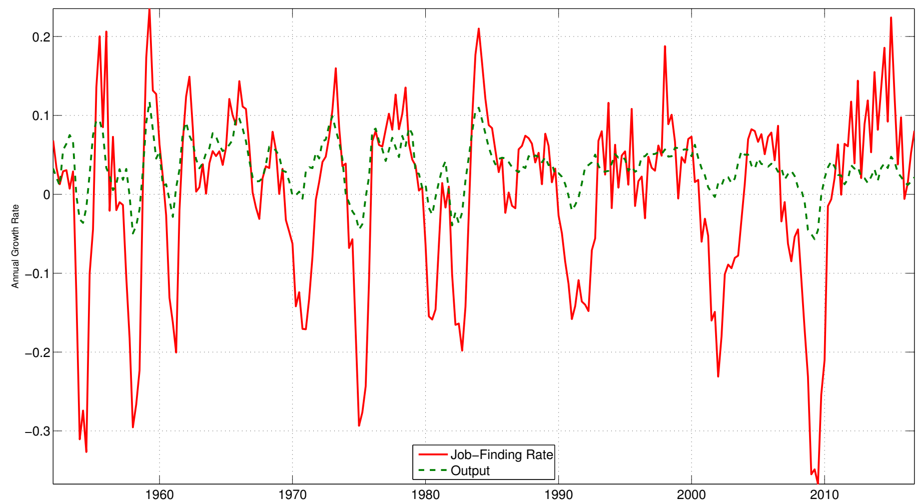

Figure 2: Annual growth rates of job-finding rate and real GDP.

by the decline of quits. Thus, the overall separation rate did not spike. Unfortunately, the JOLTS dataset only covers the period after the year 2000. We therefore focus on the role of job creation in the main part of this paper but show a version of our model with endogenous separations in the Appendix.[6]

Looking at Figure 3, the series for market tightness, i.e. the ratio between vacancies and unemployment, appears fairly symmetric. Table B.2 in the Appendix confirms that – in contrast to the other labor market variables – market tightness has a low level of skewness, which is not statistically significantly different from zero.[7]

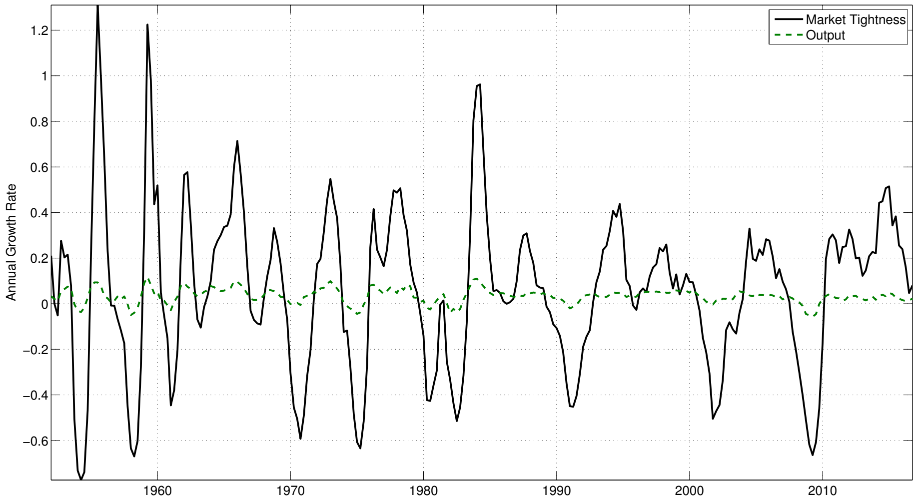

Figure 3: Annual growth rates of vacancies and market tightness.

Before we continue, let us briefly discuss the filtering method we have chosen. We have adopted the standard convention in the literature on labor market asymmetries (e.g. Abbritti and Fahr, 2013) to filter the time series from trend by looking at fourth differences in percent. However, we show in Appendix B.2 that the skewness of the job-finding rate and the unemployment rate and lack thereof in the series for market tightness are robust when looking at the percent deviation from the Hodrick-Prescott filter. Note, however, that market tightness can be artificially skewed when taking log-deviations from trend as the log-transformation generates an artificial downward skewness for time series with large fluctuations. For this reason, we prefer growth rates to log-deviations from trend as a filtering device. We discuss this in detail in Appendix B.1.

Another way to uncover asymmetries in time series is to distinguish between times

[6]Given that Ferraro (2018) shows that the participation rate is not skewed, we do not analyze this margin.

[7]Note that the asymmetry of market tightness differs in the first and second part of the observation period.

when the labor market is in a boom or a recession. For this purpose, we split the sample into periods with below or above median growth of labor market tightness. Table 1 shows the standard deviations of the annual growth rates of different labor market variables for the respective periods.

Variable	U	JFR	V	θ	Y
Tightness Growth Below Median	0.26	0.11	0.14	0.23	0.03
Tightness Growth Above Median	0.06	0.05	0.12	0.22	0.02

Table 1: Standard deviations of different labor market variables during periods when tightness growth is below and above median. Statistics are based on year-to-year growth rates for quarterly US data from 1951:I to 2016:IV.

It is evident that the job-finding rate and unemployment move a lot more during downswings than during upswings. The standard deviation of the job-finding rate and unemployment increases roughly by factor two and four respectively during downswings. By contrast, the standard deviation of market tightness is barely affected by the business cycle. The combination of all these facts allows for an interesting perspective on the matching function and the Beveridge curve.

Stylized Fact 3. *The standard constant elasticity matching function is not fully able to capture the relationship between the job-finding rate and market tightness in the data. For large market tightness growth rates (both negative and positive), the growth rate of the job-finding rate is consistently below its predicted value from a fitted Cobb-Douglas matching function.*

Figure 4 plots the annual gross growth rate of the job-finding rate against the annual gross growth rate of market tightness (in blue). For illustration purposes, we add predictions based on a nonlinear fit of a Cobb-Douglas constant returns matching function (in red). In line with the literature, we refer to the difference between the actual and predicted values as matching efficiency shifts. Note that it is common in the literature to calculate matching efficiency shifts based on a log-linear matching function estimation. However, as shown in Appendix B.1, the log-transformation affects the skewness of a time series and would therefore be misleading in our context. We take a more conservative approach and base our evaluation on a nonlinear fit of the matching function (by nonlinear least squares). Even then, the Cobb-Douglas function has trouble fitting the extremes, where it systematically overpredicts the job-finding rate. For large upswings and downswings of market tightness, most of the predicted values for the job-finding rate are above the actual realizations. The systematic difference between the job-finding rate and its predicted value suggests a decline of the matching efficiency in severe recessions if one looks at the data through the lens of a standard matching function. The apparent decline of the matching efficiency during the recent Great Recession has been widely debated in the literature (see e.g. Gavazza et al., 2018; Barnichon and Figura, 2015; Lubik, 2013).

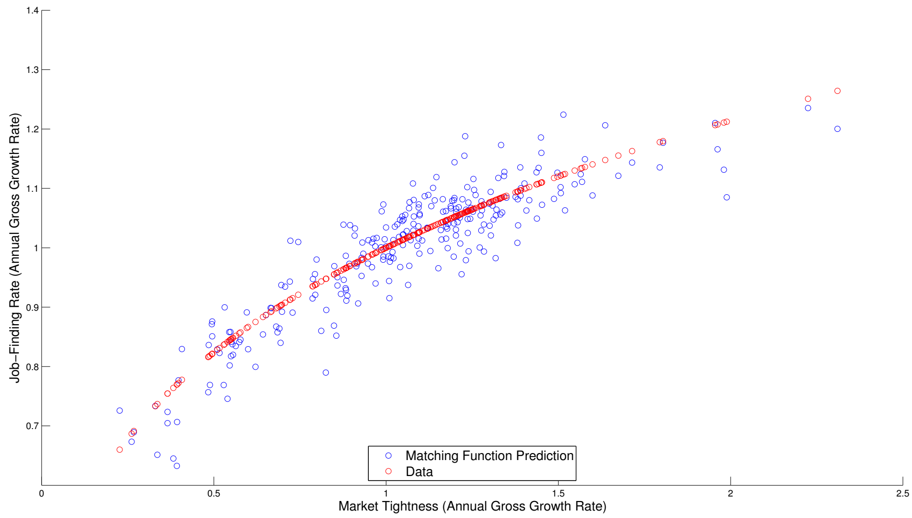

Figure 4: The US matching function (1952-2016).

Stylized Fact 4. *The path of the Beveridge curve is nonlinear over the business cycle. We provide two perspectives: First, based on annual growth rates, the Beveridge curve becomes very flat for large positive growth rates of unemployment. Second, while the Beveridge curve shows an outward shift in recessions (Diamond and Şahin, 2015), it does not show a systematic inward shift in booms.*

Figure 5 depicts the annual gross growth rates in the vacancy-unemployment-space. The relationship between vacancies and unemployment is very nonlinear. In particular, the slope of the Beveridge curve becomes nearly flat in times of high unemployment growth.

This changed comovement between unemployment and vacancies is also visible when looking at the dynamic adjustment path of vacancies and unemployment around the peak of major post-war recessions in the US. Given that we only look at a short time horizon, we do not apply any business cycle filter in this case. Figure 6 plots the vacancy and unemployment rate from four quarters before to four quarters after the peak of unemployment in each recession. This figure is a replication of Diamond and Şahin (2015) who emphasize that the Beveridge curve has shifted outward in eight out of nine recessions in the United States. We complement the work of Diamond and Şahin (2015) and repeat the same exercise for booms. Figure 7 shows the dynamic adjustment path of unemployment and vacancies around the trough of unemployment. Interestingly, in contrast to the consistent outward shift of the Beveridge curve in most recessions, there is no clear pattern in booms. Thus, the dynamic adjustment path of the Beveridge curve in recessions and in booms is clearly asymmetric.

This is an important observation. The shift of the Beveridge curve in the Great

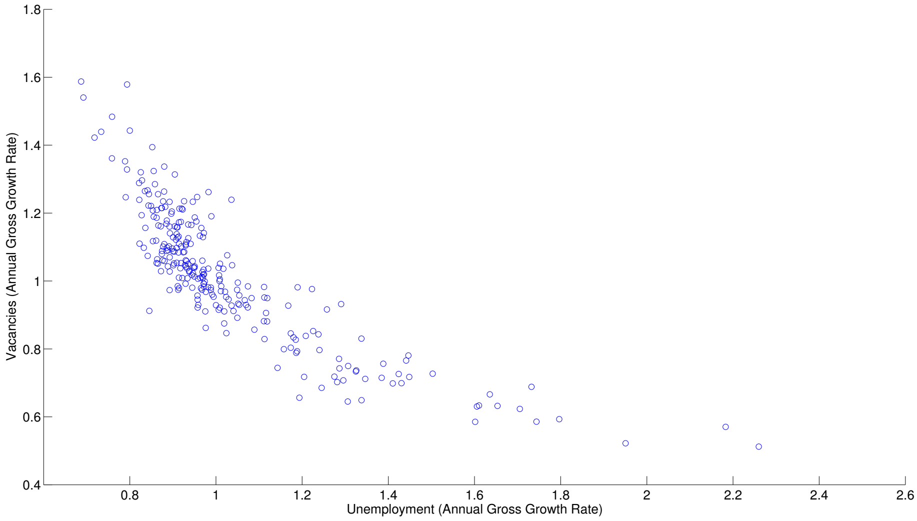

Figure 5: The US Beveridge curve (1952-2016).

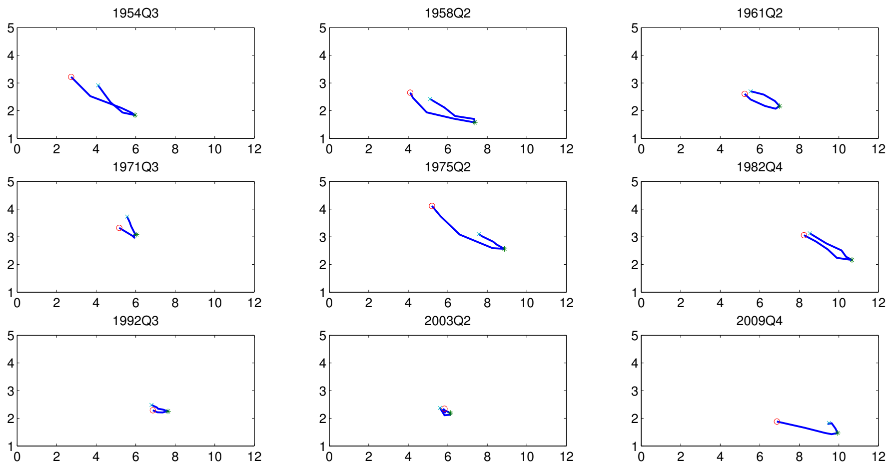

Figure 6: The US Beveridge curve in recessions. Peak of the unemployment rate +/− four quarters. The green star marks the maximum unemployment rate and the red dot (turquois cross) denotes the position four quarters before (afterwards).

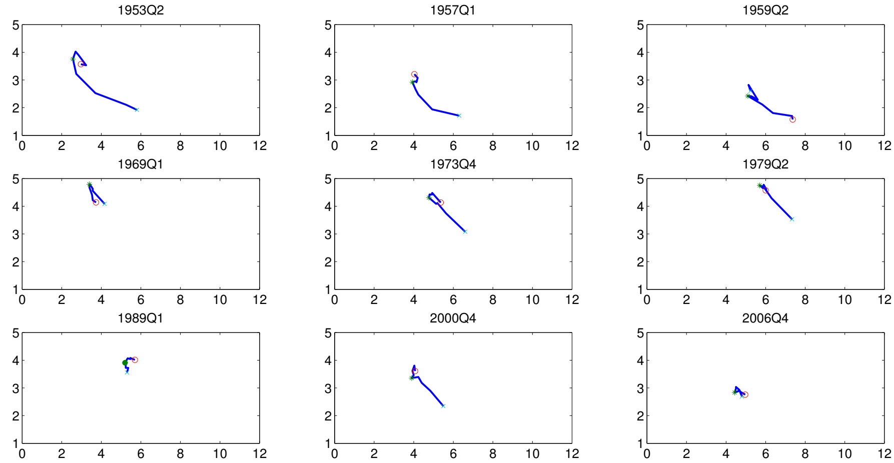

Figure 7: The US Beveridge curve in booms. Trough of the unemployment rate +/− four quarters. The green star marks the minimum unemployment rate and the red dot (turquois cross) denotes the position four quarters before (afterwards).

Recession has often been explained with a decline in matching efficiency. However, in a search and matching model, the Beveridge curve shifts outward in a recession even if matching efficiency is constant, a fact often overseen when relying on a steady state approximation for unemployment (see Christiano et al., 2015). However, in this case, we should expect a similar inward shift in booms. Figures 7 shows that this is clearly not the case suggesting a systematic departure from the stable matching function and Beveridge curve in deep recessions.

This section has collected stylized facts from the literature and complemented them with some new insights and perspective. We will show next that enhancing a search and matching model with a labor selection mechanism can explain all these facts.

3. The Model

We use a version of the Diamond-Mortensen-Pissarides (DMP) model (e.g. Mortensen and Pissarides, 1994) in discrete time and enrich it with one simple mechanism: idiosyncratic training costs for newly created jobs. Barron et al. (1989) show for the United States that firms spend on average 150 hours for on-the-job training of newly hired workers (during the first three months of employment). The reported standard deviation of training costs across firms is 207 hours (see their Table 1 in Barron et al. (1989)). This evidence shows that training costs constitute a substantial and heterogeneous cost position at the beginning of an employment spell. In addition, Barron et al. (1985, p. 50) document that "(...) most employment is the outcome of an employer selecting from a pool of job applicants (...)." In reality, there may be other sources for a heterogeneous selection behavior than training costs. One example are more general idiosyncratic productivity differentials. These are well nested in terms of our model. In Section F in the Appendix, we show results for a version of our model with idiosyncratic shocks that are persistent over the employment spell.

In our model, there is a continuum of workers on the unit interval who can either be employed or unemployed. Unemployed workers search for jobs and receive unemployment compensation b, employed workers lose their job with constant probability ϕ. Firms have to post vacancies in order to get in contact with a worker and pay vacancy posting costs κ per vacancy. We assume free-entry of vacancies. Contacts between searching workers and firms are established via a standard Cobb-Douglas contact function. In contrast to the basic search and matching model, not every contact between vacancies and workers ends in a hire. Upon contact, firms and workers draw a realization ε_{it} from an idiosyncratic training cost distribution. Only contacts with idiosyncratic training costs below a certain threshold, $\varepsilon_{it} \leq \tilde{\varepsilon}_{it}$, will transform into a job, where $\tilde{\varepsilon}_{it}$ is the cutoff training cost that makes a firm indifferent between hiring and not hiring a worker. Our model is based on Kohlbrecher et al. (2016) and similar to the stochastic job matching model (Pissarides, 2000, chapter 6) or many of the endogenous separation models (e.g. Krause and Lubik, 2007). Lechthaler et al. (2010) is an example of a labor selection model with idiosyncratic cost distributions (but without a contact function). Our model is different from the latter in assuming that only new hires vary in their idiosyncratic training costs. We choose this simplifying assumption in the main part of the paper to emphasize the role of idiosyncratic shocks for match formation. In Appendix F we show that our core results also hold in an environment with persistent idiosyncratic costs and endogenous separations.

3.1. Contacts

Contacts between searching workers and firms are established via a Cobb-Douglas, constant returns to scale (CRS) contact function,

$$c_t = \mu v_t^{\gamma} \, u_t^{s\,1-\gamma} , \tag{1}$$

where u_t^s and v_t are the number of searching workers and vacancies at the beginning of period t, μ is the contact efficiency and c_t is the overall number of contacts in period t. We define the contact probability for a worker as

$$p_t = \mu \theta_t^{\gamma}, \tag{2}$$

and the contact probability for a firm as

$$q_t = \mu \theta_t^{\gamma-1}, \tag{3}$$

with $\theta_t = v_t / u_t^s$.

3.2. The Selection Decision

Upon contact, firm and worker gain information about their match specific training cost realization at the start of the match. Technically, they draw a shock, ε_{it}, from an idiosyncratic training cost distribution, which is *iid* across workers and time, with density $f(\varepsilon)$ and cumulative distribution $F(\varepsilon)$.[8] The expected discounted profit of a firm hiring a new worker (denoted with superscript E for entrant) with match-specific training cost ε_t is given by

$$\pi_t^E(\varepsilon_t) = a_t - \varepsilon_t - w_t^E(\varepsilon_t) + \delta(1-\phi)\mathbb{E}_t\left(\pi_{t+1}^I\right), \tag{4}$$

where a_t is aggregate productivity in the economy at time t, $w_t^E(\varepsilon_t)$ is the match specific wage, δ is the discount factor, and π_t^I is the expected discounted profit of an existing match (denoted with superscript I for incumbent). Incumbent matches are not subject to training cost shocks and thus all generate the same profits. The profits are thus given by:

$$\pi_t^I = a_t - w_t^I + \delta(1-\phi)\mathbb{E}_t(\pi_{t+1}^I). \tag{5}$$

A firm will hire a worker whenever the expected discounted profit is positive. The cutoff for training costs that makes a firm indifferent between hiring and not hiring the worker is thus

$$\tilde{\varepsilon}_t = a_t - w_t^E(\tilde{\varepsilon}_t) + \delta(1-\phi)\mathbb{E}_t\left(\pi_{t+1}^I\right). \tag{6}$$

The ex-ante probability that a contact is selected into a job is thus

$$\eta_t = \int_{-\infty}^{\tilde{\varepsilon}_t} f(x)dx, \tag{7}$$

which we call the selection rate. It follows that the job-finding rate is the product of contact and selection rate:

$$jfr_t = p_t\eta_t. \tag{8}$$

3.3. Vacancies

In order to make a contact, firms have to post vacancies and pay vacancy posting costs κ. The present value of a vacancy is

$$V_t = -\kappa + q_t\eta_t\mathbb{E}_t\left[\pi_t^E|\varepsilon_t \leq \tilde{\varepsilon}_t\right] + (1-q_t\eta_t)\mathbb{E}_tV_{t+1}, \tag{9}$$

where $q_t\eta_t$ is the overall probability that a vacancy leads to a productive match. We assume free entry of vacancies such that the value of a vacancy will be driven to zero. The vacancy condition thus simplifies to

[8] Due to the *iid* assumption, we abstract from the worker-firm pair specific index i from here onward.

$$
\begin{aligned}
\frac{\kappa}{q_t \eta_t} &= E_t \left[\pi_t^E | \varepsilon_t \leq \tilde{\varepsilon}_t \right] \\
&= a_t - \frac{\int_{-\infty}^{\tilde{\varepsilon}_t} \left(x + w_t^E(x) \right) f(x)\, dx}{\eta_t} + \delta (1 - \phi) \mathbb{E}_t \left(\pi_{t+1}^I \right). \qquad (10)
\end{aligned}
$$

3.4. Wages

We assume Nash bargaining for both new and incumbent matches. Workers have linear utility over consumption. Let U_t, W_t^E, and W_t^I denote the present value of unemployment, the present value of a job for an entrant worker and the present value of a job for an incumbent worker.

$$
U_t = b + \delta \mathbb{E}_t \left(p_{t+1} \eta_{t+1} W_{t+1}^E (\varepsilon_{t+1} | \varepsilon_{t+1} \leq \tilde{\varepsilon}_{t+1}) + (1 - p_{t+1} \eta_{t+1}) U_{t+1} \right), \qquad (11)
$$

$$
W_t^E(\varepsilon_t) = w_t^E(\varepsilon_t) + \delta \mathbb{E}_t \left((1 - \phi) W_{t+1}^I + \phi U_{t+1} \right), \qquad (12)
$$

$$
W_t^I = w_t^I + \delta \mathbb{E}_t \left((1 - \phi) W_{t+1}^I + \phi U_{t+1} \right). \qquad (13)
$$

The wage for an entrant and the wage for an incumbent worker are thus determined by the following maximization problems:

$$
w_t^E(\varepsilon_t) \in \arg\max \left(W_t^E(\varepsilon_t) - U_t \right)^{\alpha} \left(\pi_t^E(\varepsilon_t) \right)^{1-\alpha}, \qquad (14)
$$

$$
w_t^I \in \arg\max \left(W_t^I - U_t \right)^{\alpha} \left(\pi_t^I \right)^{1-\alpha}. \qquad (15)
$$

3.5. Employment

The law of motion for employment is

$$
n_t = (1 - \phi) n_{t-1} + p_t \eta_t u_t^s, \qquad (16)
$$

with

$$
u_t^s = u_{t-1}, \qquad (17)
$$

and

$$
u_t = 1 - n_t. \qquad (18)
$$

3.6. Labor Market Equilibrium

Given an initial condition for unemployment u_0 and a stochastic process for technology $\{a_t\}_{t=1}^{+\infty}$, the labor market equilibrium is a sequence of allocations $\left\{ u_t, n_t, u_t^s, p_t, q_t, \pi_t, \eta_t, v_t, \tilde{\varepsilon}_t, w_t^I, w_t^E \right\}_{t=1}^{+\infty}$ satisfying equations (2), (3), (5), (6), (7), (10), (14), (15), the law of motion for employment (16), and the definitions of searching workers (17) and unemployment (18).

4. Analytical Results

4.1. Pure Selection Model

This section demonstrates that idiosyncratic training cost shocks for new contacts can generate important labor market asymmetries over the business cycle. To highlight the effect of the selection mechanism, we derive analytical results based on the steady state version of a pure selection model (see Appendix C.1 for details). We use the term "pure selection" for a model version in which the job-finding rate is solely driven by the selection rate, i.e. we assume that the contact rate for workers is constant by setting $\gamma = 0$. We therefore use the terms job-finding rate and selection rate interchangeably in this section. We prove analytically that the pure selection model generates asymmetries of the job-finding rate and thereby unemployment if the cutoff point of idiosyncratic training costs is in a downward-sloping part of the idiosyncratic distribution ($f'(\tilde{\varepsilon}) < 0$). At the same time, we show that $f'(\tilde{\varepsilon}) < 0$ is a necessary condition to obtain an empirically realistic elasticity of the job-finding rate with respect to market tightness. This corresponds to the weight on vacancies in a reduced-form matching function estimation. Thus, concave reactions of the job-finding rate over the business cycle and a plausible curvature of the matching function are two sides of the same coin in our model. In addition, we show that matching efficiency, seen through the lens of a standard search and matching model, declines under negative aggregate productivity shocks.

Result 1. *If the probability density function of idiosyncratic training costs is downward sloping at the cutoff point, the job-finding rate reacts more in recessions than in booms:* $\frac{\partial^2 \eta}{\partial a^2} < 0$ *if* $f'(\tilde{\varepsilon}) < 0$.

For the analytical proof see Appendix C.1.1.

Intuition. A negative (positive) aggregate productivity change moves the cutoff point to a part of the training cost distribution with higher (lower) density. This generates asymmetric responses of the job-finding rate. This is illustrated in Figure 8 that displays a distribution for idiosyncratic training costs along with the shift of the hiring cutoff in response to a negative and positive productivity shock. The selection rate corresponds to the area under the curve to the left of the cutoff point. Clearly, the change of the selection rate in response to a negative shock is bigger than in response to a positive shock when the hiring cutoff is situated in the downward-sloping part of the distribution.

We show analytically in Appendix C.1.1 that $f'(\tilde{\varepsilon}) < 0$ is a sufficient but not a necessary condition for generating a concave reaction of the selection rate with respect to productivity. The reason is that market tightness is convex in productivity, i.e. $\frac{\partial^2 \theta}{\partial a^2} > 0$. Under Nash bargaining, wages are a linear function of market tightness and thus also convex, i.e. $\frac{\partial^2 w^I}{\partial a^2} > 0$. Therefore, the hiring cutoff is not strictly symmetric and the selection rate could be concave even if the distribution is not downward sloping at

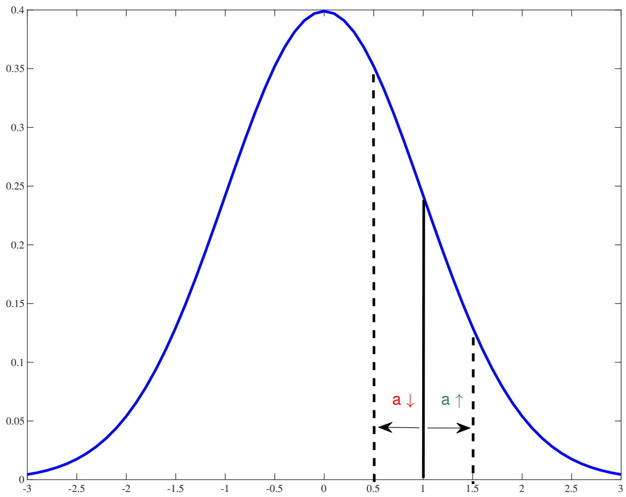

Figure 8: Idiosyncratic training costs distribution and shift of the hiring cutoff in response to positive and negative productivity shock.

the cutoff point. We show that this effect is small for plausible ranges of aggregate productivity (see Figure C.3 in the Appendix). The key driving factor for the concavity of the selection rate is the curvature of the underlying training cost distribution.

Result 2. *In a pure selection model with aggregate productivity shocks, the job-finding rate (selection rate) and market tightness show a positive comovement. The elasticity of the selection rate with respect to market tightness can be described by the following term:*

$$\frac{\Delta \ln \eta}{\Delta \ln \theta} = \frac{\partial \frac{\int_{-\infty}^{\tilde{\varepsilon}} x f(x) dx}{\eta}}{\partial \tilde{\varepsilon}} = \frac{f(\tilde{\varepsilon})}{\eta} \left(\tilde{\varepsilon} - \frac{\int_{-\infty}^{\tilde{\varepsilon}} x f(x)\, dx}{\eta} \right). \tag{19}$$

For the analytical proof see Appendix C.1.2.

Intuition. A positive aggregate productivity shock induces firms to hire workers with higher idiosyncratic training costs. Thus, the selection rate increases. At the same time, firms find it profitable to post additional vacancies. Both processes are affected by the underlying training cost distribution and the cutoff point: the former directly, the latter indirectly via the expected average training costs. This explains a positive comovement between the two variables which (in steady state) is given by the equation above. Kohlbrecher et al. (2016) show that this comovement between the selection rate and market tightness is observationally equivalent to one generated by a constant returns Cobb-Douglas matching function around the steady state. Of course, the observed link between the job-finding rate and market tightness in a pure selection model is a correlation rather than causal connection. Both the job-finding (selection) rate and tightness respond to productivity shocks but other than in a standard search and matching model, a higher tightness does not cause a higher job-finding rate.

Result 3. $\frac{\Delta \ln \eta}{\Delta \ln \theta} < 0.5$ *(generated by aggregate productivity shocks) only holds true for* $f'(\tilde{\varepsilon}) < 0$.

Equation (19) only depends on the distributional parameters of idiosyncratic training costs and the position of the hiring cutoff. We can therefore calculate the implied elasticity for a given hiring cutoff. Figure 9 illustrates this result for a number of differently shaped distributions. The upper panel shows four exemplary density functions for idiosyncratic training costs. The lower panel shows the corresponding elasticity of the job-finding rate with respect to market tightness at different cutoff points based on equation (19). Figure 9 shows that the elasticity is decreasing in the hiring cutoff. For all cases in Figure 9, the elasticity curves cross the 0.5-line after the peak of the distribution, i.e. $\frac{\Delta \ln \eta}{\Delta \ln \theta} < 0.5$ only holds for regions in which the distribution is downward sloping ($f'(\tilde{\varepsilon}) < 0$).

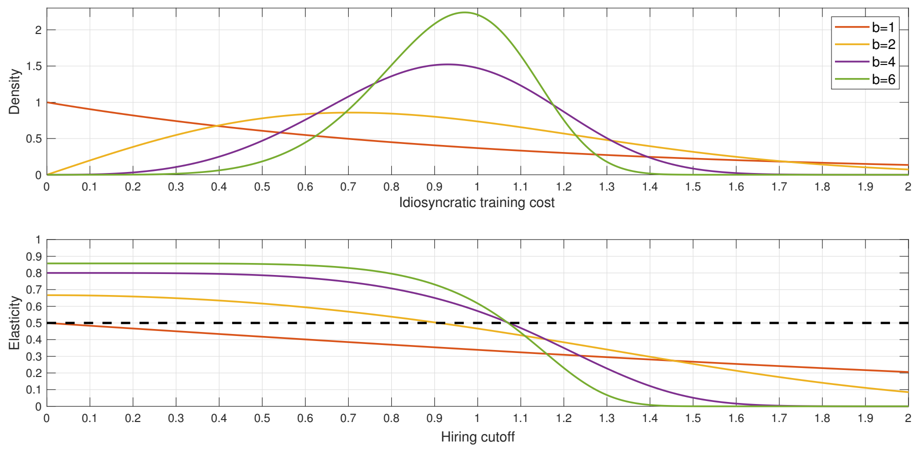

Figure 9: Density (upper panel) and corresponding elasticity of the selection rate with respect to tightness (lower panel) for Weibull distribution with scale parameter $a = 1$ and shape parameter b. Note that the shape is similar to a normal for $b = 4$, lognormal for $b = 2$, and exponential distribution for $b = 1$. The black dashed line illustrates an elasticity of 0.5.

The last result has far-reaching implications. Assume that the pure selection model is the data generating process. In this case, the elasticity of the contact rate with respect to tightness is zero and the movements of the job-finding rate are solely driven by the selection rate. Still, as shown above, the latter comoves positively with market tightness. If an empirical researcher now estimates a typical Cobb-Douglas constant returns matching function of the type $\ln jfr_t = \beta_0 + \beta_1 \ln \theta_t + \upsilon_t$, she would obtain a positive coefficient, namely $\hat{\beta}_1 = f(\tilde{\varepsilon})/\eta \left(\tilde{\varepsilon} - \int_{-\infty}^{\tilde{\varepsilon}} x f(x)\, dx/\eta\right)$ as in equation (19). This coefficient corresponds to the elasticity of the job-finding rate with respect to market tightness and the weight on vacancies in a Cobb-Douglas constant returns matching function. Typically, the empirical literature finds values of $\hat{\beta}_1$ below 0.5. This result stands out from the extensive survey by Petrongolo and Pissarides (2001) on matching function estimations and is confirmed by our own estimates.[9]

Result 4. *The condition $f'(\tilde{\varepsilon}) < 0$ is necessary for a realistic weight on vacancies in an estimated matching function ($\hat{\beta}_1 < 0.5$) and sufficient for a concave reaction of the selection rate $\left(\frac{\partial^2 \eta}{\partial a^2} < 0\right)$.*

This result directly follows from the discussion above. The estimated weight on vacancies in matching function estimations, i.e. the elasticity of the job-finding rate with respect to market tightness, is below 0.5. This is only possible if the cutoff point is in the downward sloping part of the distribution. We thus have an external validity condition for our model mechanism to generate a concave reaction of the selection rate over the business cycle.

Result 5. *With $f'(\tilde{\varepsilon}) < 0$, a negative productivity shock generates a fall of the matching efficiency if see through the lens of a standard Cobb-Douglas matching function.*

See Appendix C.1.2 for details.

Intuition. In a selection model, the elasticity of the selection (job-finding) rate varies with the position of the hiring cutoff and thus over the business cycle. This is demonstrated in Figure 9. In a recession, the hiring cutoff shifts to the left and the elasticity of the selection rate with respect to tightness increases. The opposite happens in a boom. A reduced-form constant elasticity matching function does not account for this changing relationship. From this perspective, it appears as if in recessions, i.e. high elasticity periods in our model, the job-finding rate has fallen by too much compared to the response of tightness. This shows up as a decline in matching efficiency if seen through the lens of a constant-elasticity matching function.

4.2. Combining Selection with Search and Matching

So far, we have illustrated that the labor selection mechanism can generate asymmetries of the job-finding rate in recessions. We now discuss the implications for asymmetry in the full model where both search and matching (i.e. a procyclical contact rate) and labor selection are at work.

We show in Appendix C.2 that the job-finding rate can also be asymmetric in a search and matching model without labor selection. In particular, under symmetric wages the contact rate (i.e. job-finding rate) will be concave in productivity if the elasticity of the job-finding rate with respect to market tightness (the weight on vacancies in the contact function) is below 0.5, i.e. $\gamma < 0.5$. Note that this condition shares some similarity

[9] We obtain a weight on vacancies/market tightness smaller than 0.5 independent of the filtering/detrending of the data (none, HP-filter, fourth differences, linear trend) and estimation method ((log-)linear with OLS, nonlinear with NLS). Results are available on request.

with Result 4 derived for the pure selection model. Furthermore, we show in Appendix C.2 that the job-finding (contact) rate will be the more concave, the lower the elasticity of the contact rate with respect to market tightness. Thus, a standard Cobb-Douglas contact or matching function can also generate a concave job-finding rate and the degree of concavity is closely tied to the elasticity of the contact or job-finding rate with respect to market tightness.

The elasticity of the job-finding rate with respect to market tightness thus plays an important role for labor market asymmetries in both the search and matching and the selection model. It is therefore important to understand what happens when both mechanisms are combined. As a rule of thumb, the overall elasticity of the job-finding with respect to market tightness is approximately equal to the sum of the elasticity of the contact rate and the elasticity of the selection rate (see Kohlbrecher et al., 2016, for a more detailed discussion). What does this mean for our numerical exercise? We calibrate the elasticity of the job-finding rate with respect to vacancies to 0.28. The following approximate relationship holds[10]

$$0.28 \approx \gamma + \frac{f\left(\tilde{\varepsilon}\right)}{\eta}\left(\tilde{\varepsilon} - \frac{\int_{-\infty}^{\tilde{\varepsilon}} x f\left(x\right) dx}{\eta}\right). \tag{20}$$

Compared to either a pure search and matching or a pure selection model, both terms on the right-hand side of equation (20) have to be smaller to match a given empirical target. Two implication follow: First, the contact rate is pushed to a region with larger asymmetry (as a smaller γ generates stronger asymmetries). Second, even if the estimated elasticity for the job-finding rate was above 0.5, the elasticity of the selection rate would have to be lower and thus generate the observed asymmetry of the job-finding rate. In Appendix E, we compare the ability of a standard search and matching model and the full model with selection to generate asymmetries of the job-finding rate. For a targeted amplification and a given elasticity of the job-finding rate with respect to market tightness, the model with selection generates substantially larger asymmetries.

5. Simulation and Calibration

We solve the stochastic model fully nonlinearly (see Appendix D for a description of the solution method). We perform two numerical exercises: First, we look at the impulse responses of the economy in response to a large (and persistent) aggregate productivity shock. Second, we simulate the dynamic path of the economy. Instead of drawing from a random shock series, we base our model simulation on the actual time series of labor productivity growth in the data. The advantage of this approach is that we can directly compare our simulated time series to the results in Section 2.

We follow the usual convention in the business cycle asymmetries literature and filter the data in terms of annual growth rates (see e.g. Abbritti and Fahr, 2013). This

[10]The actual elasticity is a bit smaller than the sum of the two components as there is a negative feedback effect of contacts on selection through wages.

ensures that we obtain stationary data, as required for our model simulation.[11] Thus, labor productivity – the exogenous process fed into our simulation – is defined as the annual growth rate of gross value added over employment in the nonfarm business sector from the first quarter of 1952 to the last quarter of 2016. We normalize the mean of productivity growth to one. Note that productivity itself is not skewed.[12] As we simulate on a monthly frequency, we interpolate labor productivity to a monthly frequency using industrial production and the method by Chow and Lin (1971). The simulated data is then aggregated back to quarterly frequency. The standard deviation of our quarterly productivity process (both in the data and in the simulation) is 0.02 and the autocorrelation is 0.79.

We now discuss the parametrization of our model in detail. We set the monthly discount factor to $\delta = 0.99^{1/3}$, which corresponds to a 4% annual interest rate. The empirical monthly job-finding rate and the monthly separation rate during our time span are 0.42 and 0.03 respectively. Both data series were constructed as in Shimer (2012). As our model features both large amplification and strong nonlinearities, the time series mean is different form the steady state of the model (see e.g. Hairault et al., 2010). We therefore match the time series averages of our simulation to the data – generated based on the actual productivity growth series – rather than the deterministic or stochastic steady state.[13] In addition, we target an average market tightness of one, which is a normalization. We assume that workers and firms have equal bargaining power, i.e. $\alpha = 0.5$, and that unemployed receive a compensation that amounts to 70% of aggregate productivity in steady state. Those are intermediate values between those used in Shimer (2005) and Hagedorn and Manovskii (2008).

As both the selection mechanism and contact function (see equation (20)) affect the elasticity of the job-finding rate with respect to market tightness, γ – i.e. the elasticity parameter in the contact function – must be smaller than the estimated total elasticity in the data. Kohlbrecher et al. (2016) use data on individual entry wages for Germany to determine the relative contribution of the selection mechanism and the contact function to the overall elasticity of the job-finding rate with respect to market tightness. They find that the larger part of the comovement between these variables is driven by selection. When we fit a Cobb-Douglas constant returns nonlinear matching function to the data in Section 2, we obtain an elasticity of the job-finding rate with respect to market tightness of 0.28. This is well in line with existing evidence on matching function estimations (see e.g. Petrongolo and Pissarides, 2001; Shimer, 2005). We therefore set the coefficient on vacancies in the contact function to $\gamma = 0.1$, i.e. we assume that around one third of the elasticity is driven by the contact margin.

[11] The alternative would be to take the logarithm of time series and to HP-filter them. However, as shown in Appendix B.1 this approach has caveats when using variables that exhibit large fluctuations. Tables B.3 and B.4 in the Appendix show that our results are robust to using the log-deviation or percentage deviation from the Hodrick-Prescott-Filter.

[12] If any, it would be positively skewed. However, the coefficient is very small and not statistically significant.

[13] We have targeted the deterministic steady in earlier versions of the paper. The results did not differ in any quantitatively meaningful way.

We assume a normal distribution for our match-specific training cost shocks. We determine the two parameters of the normal distribution to meet two targets from the data. The first target is the empirical elasticity of the job-finding rate with respect to market tightness, which is 0.28. Intuitively, this determines the position of the cutoff point in a given distribution (see Figure 9).[14] Given the target, it immediately follows that the cutoff point will be in the downward sloping part of the distribution. The second target is the volatility of the annualized growth rate of the job-finding rate relative to productivity in the data, which is 5.6.[15] We chose this as a target because we need a realistic volatility of the job-finding rate for a meaningful discussion about asymmetries. The second target determines how much density is required in the vicinity of the cutoff point and hence pins down the dispersion of the distribution. The resulting standard deviation of our distribution is 0.11. The average selection rate in the simulated model is 0.94 and the average contact rate is 0.45. Given the target for market tightness, we obtain a contact efficiency of $\mu = 0.45$. Finally, we set the vacancy posting cost to satisfy the zero profit condition.

Two comments are in order regarding the distributional choices. First, the focus of this paper is not the ability of the selection mechanism to generate amplification.[16] Instead, we analyze the business cycle properties of a search and matching model with selection that features a realistically volatile job-finding rate. This is a prerequisite for quantitatively meaningful nonlinearities. Choosing a small dispersion of the idiosyncratic training cost distribution (in our case a standard deviation of 0.11) helps us to achieve the required amplification. An alternative strategy would be to choose a more dispersed idiosyncratic training cost shock distribution and to apply a different strategy to generate amplification of the model (e.g. sticky wages). Intuitively, it does not matter for our purposes whether a given change in the selection rate is driven by a small change of the cutoff in an area with high density or by a large change of the cutoff in an area with low density. Second, what would happen if we chose a different than normal distribution? For an elasticity of the selection rate with respect to market tightness strictly smaller than the estimated elasticity of the job-finding rate, the cutoff point would always lie in a downward-sloping part of the distribution. This is demonstrated in Figure 9 for a number of distributions with very different shapes.[17] Hence, the model would always generate left-skewness in the selection rate which would ceteris paribus translate into left-skewness of the job-finding rate. However, depending on the distribution, there would be higher or lower density at that point which again might require a different mechanism to generate amplification.

Note finally that our wage is fairly flexible. For incumbent wages and for a given real-

[14]The corresponding picture for the normal distribution looks very similar to the case of the Weibull distribution with shape parameter $b = 4$.

[15]Note that in the model these moments are based on the percentage deviation from the stochastic steady state as the productivity series is already denoted in growth rates.

[16]See Chugh and Merkl (2016) for a discussion of the ability of selection models to generate amplification.

[17]This would of course not be the case for a uniform distribution, which by construction is flat. Given that there is usually a high degree of curvature in residual entry wages (see e.g. Kohlbrecher et al., 2016), we disregard this possibility.

ization of idiosyncratic training costs, a one unit increase in productivity (corresponding to one percent starting from the steady state) leads to a 0.89 unit increase in wages. The mean entry wage, however, only increases by 0.69 units. The reason is a composition effect: If productivity goes up, firms are willing to hire workers with higher training costs, which lowers the average entry wage.

6. Results

6.1. Inspecting the Mechanism Numerically

To illustrate the driving forces for business cycle asymmetries and the shift of the matching efficiency, Figure 10 shows the fully nonlinear impulse response functions of the model economy to a 2.5% positive and a 2.5% negative productivity shock. These exercises are meant to replicate severe recession and boom scenarios. Although the aggregate shocks have the same absolute size, the job-finding rate and the unemployment rate respond very asymmetrically. The job-finding rate drops by up to 16% and unemployment increases by a maximum of 14% in response to the negative shock. By contrast, the job-finding rate increases by only 7% and the unemployment rates falls by 5% in response to the positive aggregate productivity shock. Market tightness, in contrast, moves almost symmetrically.

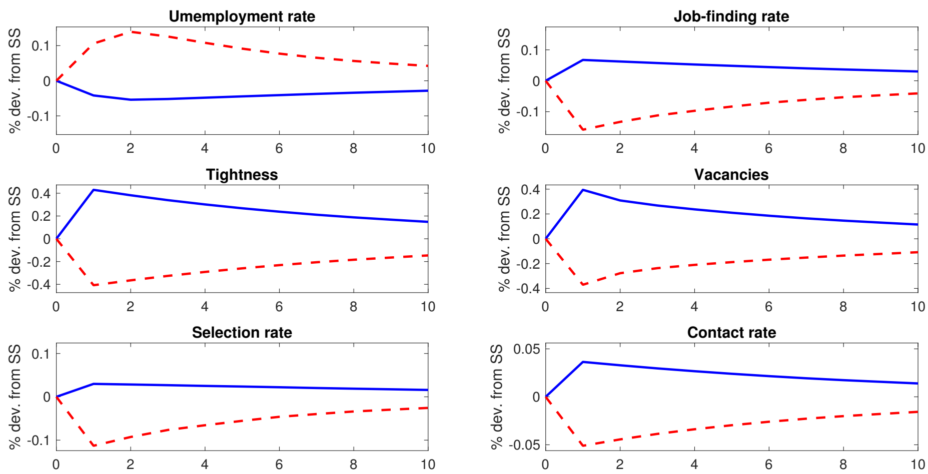

Figure 10: Model impulse responses to a 2.5% positive (solid blue line) and a 2.5% negative (dashed red line) productivity shock (in percent deviations from steady state).

This exercise illustrates that the nonlinearities generated by the selection part of the model are quantitatively meaningful. Given that the cutoff point is located at a downward sloping part of the density function in steady state (i.e. $f'(\tilde{\varepsilon}) < 0$), a negative aggregate shock moves the selection cutoff point to a part of the idiosyncratic training cost distribution with higher density. Thus, a given change of the cutoff point exerts a larger effect on the selection and hence job-finding rate. While the response of the contact rate is also somewhat stronger in the recession scenario, Figure 10 clearly shows that the selection rate is the driving force behind the asymmetric response of the job-finding rate. This is further illustrated in Appendix E where we compare our model to a search and matching model without a selection mechanism.

While the job-finding rate falls a lot in a severe recession, the response of market tightness is muted. Firms anticipate in their vacancy posting behavior that they hire workers with on average lower idiosyncratic training costs in a recession compared to a boom. This composition effect mutes the response of vacancies and market tightness to an aggregate productivity shock.

6.2. Beveridge Curve and Matching Function

Figure 11 shows the dynamic response of the model economy in response to a 2.5% positive and negative productivity shock in the unemployment-vacancy space. In our simulation exercise, vacancies are fairly symmetric, moving by roughly 39 and 37 percent (in absolute terms) during the boom and during the recession. By contrast, due to the nonlinearities of the model, unemployment moves by about one half more in recessions compared to booms. As in the data, the recession is associated with a strong outward shift of the Beveridge curve. By contrast, the inward shift in the boom scenario is less pronounced. Thus, our model provides an explanation for the differential response of the Beveridge curve in booms and recessions observed in the data.

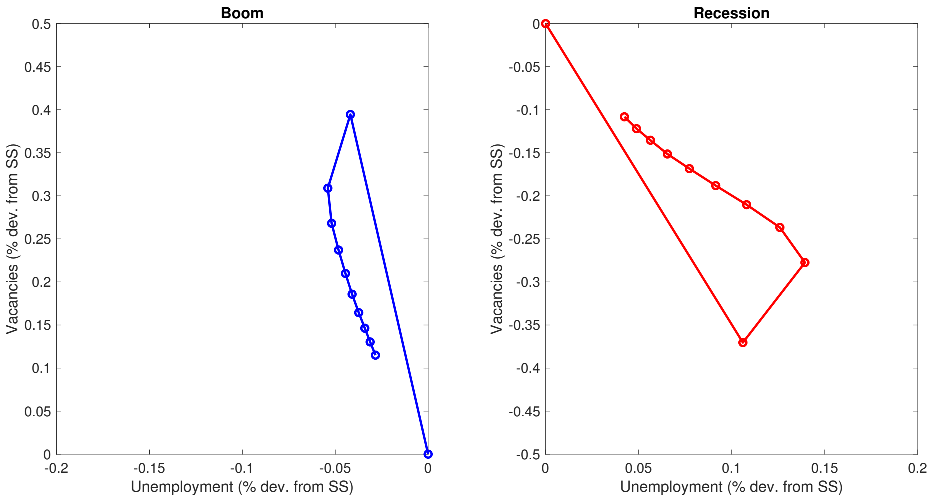

Figure 11: Beveridge curve shift in boom and recession scenario. Model response to a 2.5% positive (left panel) and 2.5% negative (right panel) productivity shock.

This phenomenon can be illustrated further by looking at the Beveridge curve for the dynamic simulation of the model, which is based on the actual productivity growth process for the US. Figure 12 shows that the simulated Beveridge curve has a particularly nonlinear shape, i.e. becomes very flat, in times of high unemployment. This is again very much in line with the corresponding figure based on US data in Section 2.

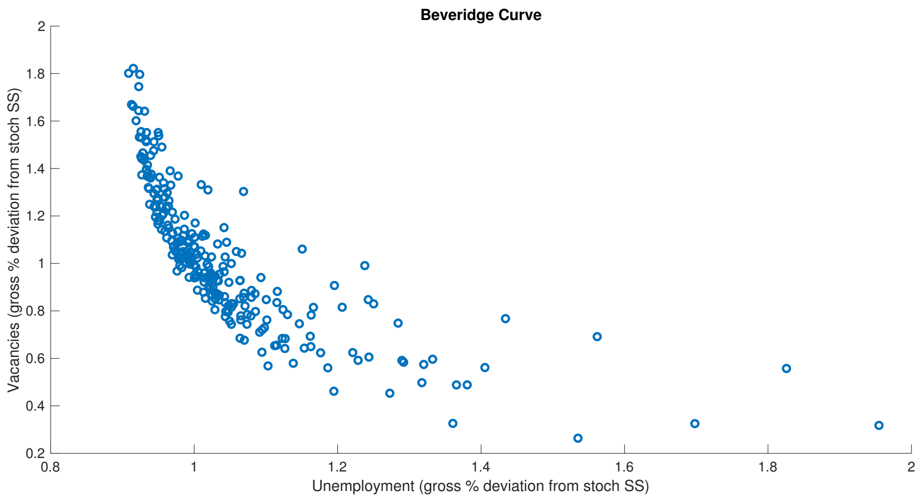

Figure 12: The model-based Beveridge curve.

This nonlinearity is also apparent when looking at the model through the lens of a standard matching function. Figure 13 plots the model generated job-finding rate and market tightness along with the predicted values from a nonlinear fit of a Cobb-Douglas matching function as in Figure 4 from Section 2. The constant-elasticity matching function systematically overpredicts the job-finding rate in times of very low and very high market tightness. In Section 4, we explained the driving forces for this mechanism. In severe recessions, the job-finding rate drops by a lot due to the nonlinearities generated by the idiosyncratic training cost density function. By contrast, the response of market tightness is muted. Technically, when the hiring cutoff shifts to the left in response to a negative shock, the relationship between the job-finding rate (through selection) and tightness in the model shifts which is reflected in an increase of the elasticity between these variables (see Figure 9). Thus, through the lens of a constant-elasticity matching function, it appears as if the job-finding rate has dropped by too much.[18] This is interpreted as a decline of the matching efficiency, i.e. an exogenous deterioration of the efficiency of the labor market. However, through the lens of our model, this decline is simply a result of the severe recession, the labor selection mechanism and the resulting nonlinearities. Once the aggregate shock disappears, the selection rate will return to its steady state level and matching efficiency will recover.

[18] The matching efficiency also drops in strong booms in our numerical simulation. In a boom, the elasticity of the job-finding rate with respect to tightness falls. Thus, a constant elasticity matching function overpredicts the response of the job-finding rate in a strong upswing.

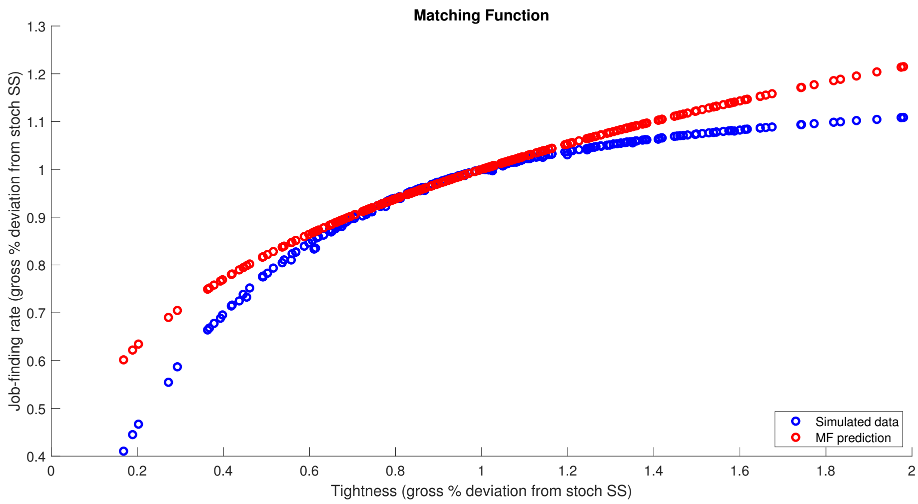

Figure 13: The model-based matching function.

6.3. Matching Efficiency in the Great Recession

Using the example of the Great Recession, we now demonstrate that the endogenous drop of matching efficiency in our model is quite substantial in severe recessions. Given that our model is only driven by productivity shocks, our simulated job-finding rate unsurprisingly does not fully capture the dynamics of the job-finding rate during that period. We therefore ask the following counterfactual question: If the job-finding rate in our model did take the same trajectory as the actual job-finding rate during the Great Recession, by how much would the matching efficiency drop? For this purpose, we fit the simulated job-finding rate to the actual annualized quarterly growth rate of the job-finding rate between 2007 and 2011.[19] Both series are displayed in Figure 14.

Next, we back out the model-implied matching efficiency drop by looking at the deviation between the simulated job-finding rate and the predicted job-finding rate based on the matching function we have estimated on the full sample simulation in Section 5.[20] The corresponding series is displayed as the dashed-dotted line in Figure 14 along with the matching efficiency drop obtained from the real data (dotted line). While the simulated matching efficiency is close to its predicted value at the beginning and end of the period (i.e. the deviation is zero), it drops by more then 12 percent in the course of the recession. Thus, for a realistic labor market slump, our model is able to explain a large part of the observed decline of matching efficiency in the data. In addition, our

[19]More precisely, we first interpolate the quarterly series of the job-finding rate to a monthly frequency and then search for the nearest neighbor in our solved model. As both tightness and the selection rate are jump variables, we have a unique job-finding rate associated with every aggregate state and thereby get a sequence of states to feed into the simulation. See Appendix D for details on the solution method.

[20]Note that the elasticity of the job-finding rate with respect to market tightness was targeted to match its empirical counterpart.

model tracks the overall trend of the empirical matching efficiency quite well although the trajectory of the of matching efficiency in the model is smoother and the recovery too quick compared to the data. Note that in line with our previous exercises, we have fitted the job-finding rate in the model to the annualized growth rate of the job-finding rate in the data. We obtain a similar picture if we fit the level of the job-finding rate during the Great Recession instead.[21]

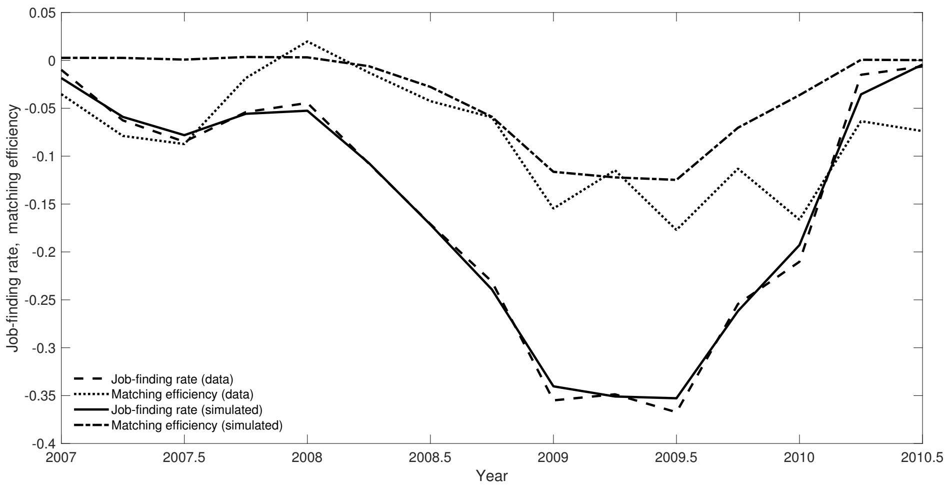

Figure 14: Actual (in annualized growth rates) and simulated job-finding rate (in percent deviation) along with data and model-implied matching efficiency drop during the Great Recession.

Obviously, in recessions there are other factors at work that lead to a decline of the matching efficiency such as mismatch (see Şahin et al., 2014). However, these factors were only found to explain a certain part of the actual decline of the matching efficiency during the Great Recession. Thus, we view our mechanism as a complementary explanation. In particular, while our model is well able to explain the initial drop of the matching efficiency, other mechanism might be better suited to explain its slow recovery.

6.4. State Dependent Effects

To illustrate the state dependency of the labor market, we use the simulated labor market time series – based on the real productivity growth series – and calculate their standard deviations for periods when simulated labor market tightness is below and above the median. Table 2 shows that the job-finding rate and unemployment fluctuate a lot more during downswings than during upswings. This corresponds to the pattern in US data, as illustrated in Section 2.

The nonlinearities generated by the selection mechanism do not only create asymmetric business cycles in the labor market, but also matter for the effectiveness of policy

[21]Results are available upon request.

Variable	U	JFR	V	θ	Y
Tightness Below Median	0.16	0.12	0.16	0.19	0.03
Tightness Above Median	0.03	0.03	0.20	0.24	0.02

Table 2: Standard deviations of different labor market variables during periods when tightness is below and above median.

interventions. To illustrate this point, we assume that the government implements a wage subsidy of 1% of productivity for all new and incumbent worker-firm pairs to stimulate the economy. The subsidy program stays in place for half a year and is financed by lump-sum taxation. It thus has no distortionary side effects.

When the government implements the subsidy, the present value of a job increases, vacancy posting rises and the selection rate goes up. To illustrate the nonlinearities, we show how the effects of the policy differ when it is implemented at the beginning of a recession and at the beginning of a boom (corresponding to a 2.5% persistent productivity shock). Figure 15 shows that the effect of the policy on the job-finding rate and

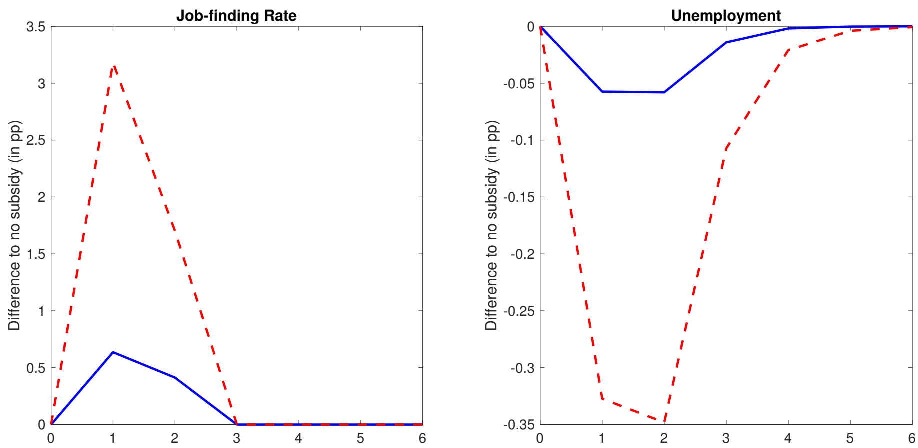

Figure 15: Reaction of unemployment and the job-finding rate in response to a wage subsidy during boom (blue solid line) and recession (red dashed line). Deviation (in percentage points) from boom and recession scenario without wage subsidy.

unemployment is several times larger during the recession than during the boom. The intuition is straightforward: The selection cutoff point is at a part of the idiosyncratic training cost distribution with higher density during the recession. Thus, the government intervention has a larger effect because a given change in the present value moves the selection rate and thereby the job-finding rate and unemployment by more.

We have picked a simple wage subsidy with lump-sum financing for illustration purposes and to keep the model simple. However, our model framework predicts that any policy intervention that affects the net present value of a match and thereby the selection mechanism has stronger employment effects during recessions than during booms. This also holds for government spending if it affects the relative price of labor. This is indeed the case if we embed our framework into a New Keynesian setting.[22] The state-dependent effects of fiscal policy are in line with evidence such as Auerbach and Gorodnichenko (2012).

Obviously, this section provides only positive statements. However, Chugh et al. (2018) show that an optimal Ramsey planner would choose countercyclical hiring subsidies under the presence of labor selection. Thus, time varying fiscal interventions that affect the present value of matches can be welfare increasing. We leave a detailed analysis of the interaction between the identified nonlinearities and normative issues for future research.

7. Connection to the Literature

Our paper is connected to the literature on business cycle asymmetries and to the literature on the decline of matching efficiency/shifts of the Beveridge curve in recessions. This section provides a brief praise of the most relevant papers.

Business cycle asymmetries have been a topic in the empirical literature already a long time ago (e.g. Long and Summers, 1986; Neftçi, 1984). An early theoretical explanation of unemployment asymmetries was provided by Huizinga and Schiantarelli (1992) using a dynamic insider-outsider model with endogenous reservation utility. More recently, McKay and Reis (2008) show that contractions in the United States are briefer and more violent than expansions. They propose a model with asymmetric employment adjustment costs and a choice when to replace old technologies to account for these facts. Abbritti and Fahr (2013) show for various OECD countries that unemployment is skewed over the business cycle. They are able to explain the unemployment asymmetries with a model of asymmetric wage adjustment costs (i.e. nominal wage cuts are more costly than wage increases). While the asymmetry of (un)employment is well known, we document that the job-finding rate is very skewed in the United States. We are the first to propose idiosyncratic training cost shocks to be the driving source for this phenomenon.

Search and matching models without idiosyncratic shocks for match formation are also able to generate asymmetries of the job-finding rate. As already noted, Abbritti and Fahr (2013) assume asymmetric wage adjustment costs. Petrosky-Nadeau and Zhang (2017), Petrosky-Nadeau et al. (2018), and Ferraro and Fiori (2018) all use calibrations in the spirit of Hagedorn and Manovskii (2008). In these models, this is needed to generate enough amplification of the job-finding rate. Our model differs in several ways: First, the asymmetry is predominantly driven by the selection rate, or more precisely the curvature of the training cost distribution. Thus, we provide an additional complementary mechanism for generating asymmetries of the job-finding rate. Second, the selection mechanism can also explain why matching efficiency drops in recessions. Per definition this cannot be accounted for in models in which the job-finding rate moves along a stable matching function. Finally, as the selection mechanism provides additional amplification, we neither require wage rigidity nor a small surplus calibration in order to generate large

[22]Results are available on request.

movements of the job-finding rate (see Chugh and Merkl (2016) for an explanation of the economic mechanism).

Ferraro (2018) shows that the Mortensen and Pissarides (1994) framework can be modified to generate labor market asymmetries. In his model, workers have permanently different skills and search on distinct submarkets. A negative shock causes a spike in separations in the low-skilled submarket which worsens the quality of the unemployment pool and depresses job findings. Cairó and Cajner (2018) show that the separation rates for different skill groups are indeed remarkably different, both in terms of the level and the dynamics (see their Figure 2). However, Cairó and Cajner (2018) also show that the job-finding rate for all skill groups shows a strong and rapid decline during the Great Recession. Our model offers an explanation for this asymmetry within skill groups. Thereby, it is complementary to Ferraro (2018).

The debate on shifts of the matching efficiency was spurred by the Great Recession. Diamond and Şahin (2015) show that Beveridge curve shifts occurred during most post-war recessions in the United States. Barnichon and Figura (2015) discuss reasons for the procyclicality of the matching efficiency (in particular during the Great Recession). Barnichon and Figura (2015) also use heterogeneities as an explanation for shifts of the Beveridge Curve. However, in contrast to us they use systematic differences in search efficiency and labor market segmentation as driving forces, while we use idiosyncratic shocks for match formation. Thus, their theoretical explanation is complementary to ours. Gavazza et al. (2018) explain the decline of matching efficiency during the Great Recession with a decline in recruiting intensity. While the quantitative implications for matching efficiency are similar, their mechanism is very different from ours. Finally, none of the discussed papers connects the decline of matching efficiency and the shift of the Beveridge curve to the asymmetric movement of the job-finding rate. This is also true for Sedláček (2014) who is closest to our paper in terms of the mechanism. He also shows that matching efficiency is procyclical and proposes a model with idiosyncratic productivity shocks for both new matches and incumbent workers and firing costs to account for this. Our paper is different in several dimensions: First, we focus on the nonlinear structure of the model, while Sedláček (2014) linearizes the model in order to estimate it. This allows us to also discuss business cycle asymmetries and connect them to shifts of the matching efficiency. Sedláček (2014) estimates a model without idiosyncratic shocks and then calibrates the model with idiosyncratic shocks. We show that idiosyncratic shocks for match formation change the elasticity on vacancies/market tightness in the estimated matching function. In other words, if idiosyncratic shocks matter for match formation, the elasticity of matches with respect to vacancies is not the same as the coefficient in the theoretical Cobb-Douglas matching function. Even if this is taken into account, there are additional shifts of the matching efficiency that are due to nonlinearities.

8. Conclusion

Unemployment and the job-finding rate in the United States are strongly asymmetric over the business cycle. The movement of these variables is stronger in recessions than in booms. In addition, matching efficiency as seen through the lens of a standard matching function seems to fall in recessions. This is related to the outward shift of the Beveridge curve during those times. In our paper, we show that idiosyncratic training costs shocks for new matches (within an otherwise standard search and matching framework) can explain these empirical facts. We argue that both the asymmetric movement of the job-finding rate and the apparent decline of matching efficiency during severe recessions are two sides of the same coin driven by the selection behavior of firms over the business cycle.

In our model, due to idiosyncratic training cost shocks only a fraction of workers meeting a firm is selected into jobs and the selection rate fluctuates asymmetrically over the cycle. This drives the observed skewness of the job-finding rate. We show analytically that the selection rate falls more in recessions than it increases in booms if the hiring cutoff for idiosyncratic training cost realizations is situated in a downward sloping part of the training cost distribution. We further prove that this condition is always met for empirically plausible values of the elasticity of the job-finding rate with respect to market tightness. Quantitatively, the reaction of the job-finding rate in our model is substantially larger in response to a negative shock compared to a positive shock. Our results are robust when we add endogenous separations.

We further show that the asymmetric nature of the selection rate implies that the elasticity of the job-finding rate with respect to market tightness is no longer constant over the business cycle. When the data is seen through the lens of a standard constant returns matching function, we observe a drop in matching efficiency in severe recessions. We show that our mechanism can account for an important part of the decline of matching efficiency in the Great Recession in the United States.

Finally, the asymmetric nature of the selection mechanism can also rationalize why certain government interventions can be expected to have time varying effects (depending on the business cycle state). We leave the integration of this mechanism into a larger-scale model and a detailed normative analysis for future research.

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A. Data

We use quarterly data from 1951 to 2016 that translate into annual growth rates from 1952 to 2016. We start in 1951 as this is the earliest available date for vacancies constructed by Barnichon (2010).

Table A.1: Data

Variable	Description	Source
Output	Real Gross Domestic Product in 2005 Dollar	NIPA-tables (FRED)[1]
Job-finding rate	Job finding probability for unemployed workers	BLS (FRED) – Calculation as in Shimer (2012)
Unemployment rate	Unemployment Rate	BLS (FRED)
Vacancies	Composite Help-wanted Index	from Barnichon (2010)[2]
Vacancy rate	Constructed as in Diamond and Şahin (2015)[3]	from Barnichon (2010), JOLTS, BLS (FRED)
Industrial Production	Industrial Production Index	Federal Reserve Board (FRED)
Labor Productivity	Real Gross Value Added: GDP/Employment in non-farm business sector	BEA/BLS (FRED)

[1] FRED: Federal Reserve Bank of St. Louis Economic Database (`http://research.stlouisfed.org/fred2/`)

[2] Barnichon's Composite Help-wanted Index (`https://sites.google.com/site/regisbarnichon/research`)

[3] Composite Help-wanted Index matched to JOLTS vacancies in December 2000, JOLTS vacancies from 2001 onwards; time series divided by civilian employment.

B. Skewness and Filtering

We have adopted the standard practice in the business cycle asymmetries literature (e.g. Abbritti and Fahr, 2013) to calculate annual growth rates of various aggregate labor market time series in percent deviations. When discussing asymmetries, taking the percentage change of the fourth differences is a sensible choice for retrieving the cyclical deviation of a time series from its trend. Table B.2 shows skewness numbers for the main series of interest filtered this way.

In the following, we explain why we do not use log-deviations from trend and what would change if we used the Hodrick-Prescott filter instead of fourth differences.

Variable	JFR	U	V	θ	Y
Skewness	-0.91	2.26	-0.12	0.14	-0.34
P-Value	0.01	0.05	0.26	0.32	0.03

Table B.2: Business Cycle Skewness. Statistics are based on year-to-year growth rates for quarterly US data from 1951 to 2016. Test statistics for skewness follow Bai and Ng (2005).

B.1. Growth Rates versus Log-Approximation

In contrast to other aggregate variables (such as productivity or GDP), labor market variables exhibit very large fluctuations. As a consequence, the approximation of growth rates with the fourth log-differences is not suitable, as illustrated by Figure B.1.

The approximation mistake is particularly extreme for market tightness, which shows the largest movement of all variables. While the actual annual growth rate of market tightness ranges in between -77 percent and 131 percent, the log-approximation ranges in between -148 percent (which is impossible by definition) and 84 percent. The bottom line is that the log-approximation artificially skews the labor market growth rates downwards.

This would bias our results on nonlinearities from Section 2. Given that the job-finding rate fluctuates a lot less than market tightness, the bias is particularly big for the latter. As a consequence, the negative log growth rates of the market tightness become a lot larger and the matching function appears to be almost linear (compare the approximation in the upper panel of Figure B.2 to the real growth rates in the lower panel). However, this is just the result of the approximation mistakes. The same is true for the Beveridge curve. Thus, this Appendix sounds a cautionary note on using log-approximation for labor market variables because this may generate misleading results in terms of labor market asymmetries.

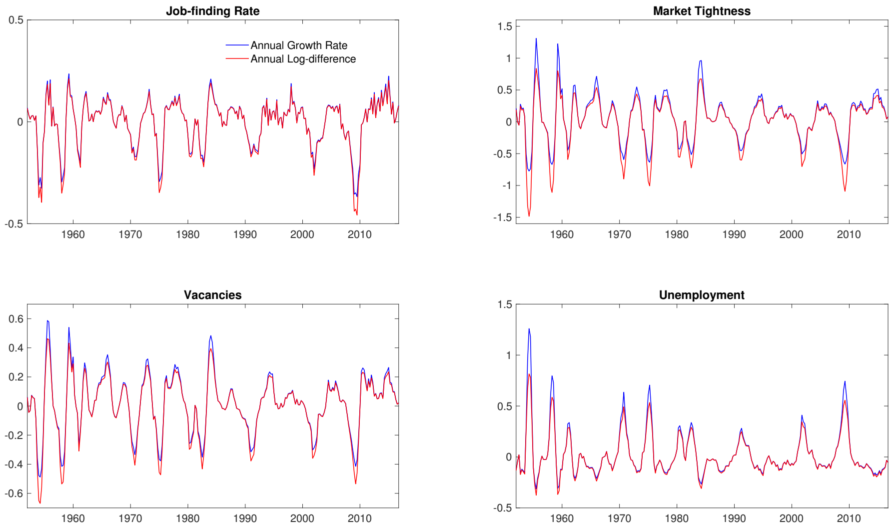

Figure B.1: Annual growth rates in percent (blue line) and in log-differences (red line) for different time series.

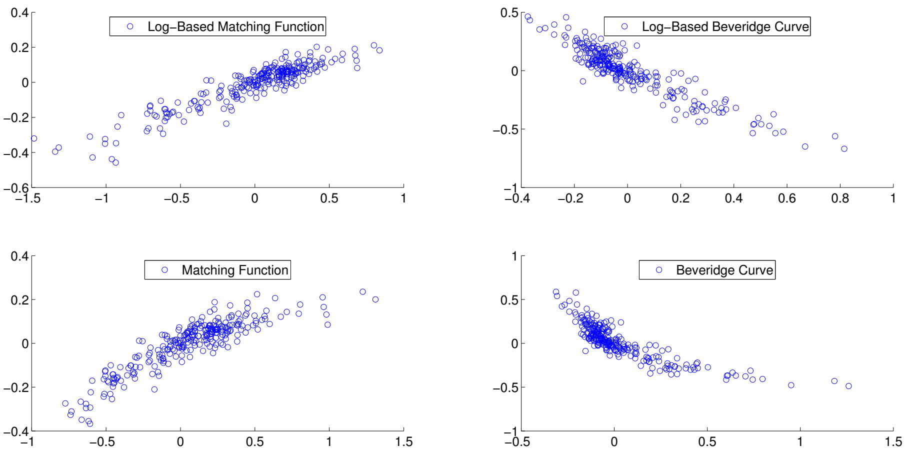

Figure B.2: Annual growth rates in percent and in log-differences for Beveridge curve and matching function.

B.2. Hodrick-Prescott-Filter

Our critique also applies when using the Hodrick-Prescott (HP) filter for logarithmic time series (Hodrick and Prescott, 1997). To illustrate this point, Table B.3 shows skewness numbers for the standard procedure to apply the Hodrich-Prescott filter (with smoothing parameter $\lambda = 1600$) to logarithmic variables and to calculate log-deviations from trend. By contrast, Table B.4 is based on the unusual procedure[23] to calculate the HP-filter based on levels and to show percent deviations from this HP-trend. These two procedures deliver strikingly different results for some of the labor market variables. In all cases, the skewness measure is lower for the log-case. As argued before, this is because the log biases the skewness downward. This problem is particularly severe for those series with the largest fluctuations. The skewness for market tightness is -0.49 and statistically significant for the log-deviations but only -0.08 and insignificant based on percent deviations.

Variable	JFR	U	V	θ	Y
Skewness	-0.33	0.31	-0.58	-0.49	-0.43
P-Value	0.02	0.07	0.02	0.03	0.07

Table B.3: Business Cycle Skewness. Statistics are based on log-deviations from Hodrick-Prescott filter with smoothing parameter $\lambda = 1600$ for quarterly US data from 1951 to 2016. Test statistics for skewness follow Bai and Ng (2005).

Variable	JFR	U	V	θ	Y
Skewness	-0.27	0.52	-0.35	-0.08	-0.32
P-Value	0.04	0.06	0.03	0.18	0.10

Table B.4: Business Cycle Skewness. Statistics are based on percent-deviations from Hodrick-Prescott filter (based on levels) with smoothing parameter $\lambda = 1600$ for quarterly US data from 1951 to 2016. Test statistics for skewness follow Bai and Ng (2005).

As a second critical remark, the HP filter itself may generate a skewed trend and thereby mask parts of the skewness of the underlying time series. This would certainly be true for the job-finding rate in the Great Recession, with a violent and asymmetric decline of the job-finding rate that remained below its long-run trend for several years. The HP-filter would smoothly adjust downwards and thereby absorb part of the skewed behavior, which would then be part of the trend and not the cycle.

[23] Although this procedure is unusual, $hpfilter\,(\log(\theta))$ and $(hpfilter\,(\theta))$ deliver a correlation of 0.97. If we logged the latter, we would obtain a correlation close to 1. Thus, the trend movement is changed very little by this procedure.

Due to these concerns (artificial skewness of the logarithm and skewness of the trend), we prefer to show the time series based on annual growth rate (fourth differences) as shown in Table B.2. To us, this appears to be the more innocuous option to remove the trend from the data than the HP-filter. However, even if we used the HP-filter as our baseline version, the key facts that are important for our paper remain stable. We find a leftward skewed job-finding rate and a rightward skewed unemployment rate. In addition, the matching efficiency, seen through the lens of a standard matching function, shows a procyclical pattern.

C. Analytical Appendix

We have derived a search and matching model with labor selection in the main part. To illustrate the respective role of the selection and contact margin for asymmetries, we separate our model into two analytical parts. First, we analyze the role of nonlinearities and the matching efficiency in a selection model with a degenerate contact function ($\gamma = 0$). Second, we do the same for a search and matching model without labor selection.

C.1. Selection Model

In steady state, a selection model can be described by four equations: the wage equation, the cutoff point, the selection rate, and the vacancy free-entry condition.

Let us assume that the wage for an entrant worker at the hiring cutoff is given by

$$w^E(\tilde{\varepsilon}) = w^I - \alpha\tilde{\varepsilon}, \tag{21}$$

where w^I is the wage for incumbent workers. The cutoff point then satisfies:

$$\tilde{\varepsilon} = \frac{a - w^I}{(1 - \delta(1 - \phi))(1 - \alpha)}. \tag{22}$$

The steady state selection rate is

$$\eta = \int_{-\infty}^{\tilde{\varepsilon}} f(x)\,dx, \tag{23}$$

and steady state market tightness is

$$\begin{aligned} \theta &= \frac{p\eta}{\kappa}\left(\frac{a - w^I}{1 - \delta(1 - \phi)} - \frac{(1-\alpha)\int_{-\infty}^{\tilde{\varepsilon}} x f(x)\,d\varepsilon}{\eta}\right) && (24) \\ &= (1-\alpha)\frac{p\eta}{\kappa}\left(\tilde{\varepsilon} - \frac{\int_{-\infty}^{\tilde{\varepsilon}} x f(x)\,dx}{\eta}\right). && (25) \end{aligned}$$

C.1.1. Business Cycle Asymmetries

The first derivative of the selection rate is

$$\frac{\partial \eta}{\partial a} = f\left(\tilde{\varepsilon}\right) \frac{\partial \tilde{\varepsilon}}{\partial a} > 0, \tag{26}$$

and the first derivative of the hiring cutoff is

$$\frac{\partial \tilde{\varepsilon}}{\partial a} = \frac{1 - \frac{\partial w^I}{\partial a}}{\left(1 - \delta\left(1 - \phi\right)\right)\left(1 - \alpha\right)}. \tag{27}$$

Given that $\frac{\partial w^I}{\partial a} < 1$ for plausible wage formation mechanisms, $\frac{\partial \tilde{\varepsilon}}{\partial a} > 0$, i.e. higher aggregate productivity leads to more hiring.

Business cycle asymmetries can be detected by looking at the second derivative of the selection rate with respect to productivity changes:

$$\frac{\partial^2 \eta}{\partial a^2} = f'\left(\tilde{\varepsilon}\right) \left(\frac{\partial \tilde{\varepsilon}}{\partial a}\right)^2 + f\left(\tilde{\varepsilon}\right) \frac{\partial^2 \tilde{\varepsilon}}{\partial a^2}. \tag{28}$$

In order to understand this equation better, we need to look at the second derivative of the cutoff point with respect to productivity, which is

$$\frac{\partial^2 \tilde{\varepsilon}}{\partial a^2} = \frac{-\frac{\partial^2 w^I}{\partial a^2}}{\left(1 - \delta\left(1 - \phi\right)\right)\left(1 - \alpha\right)}. \tag{29}$$

Insight 1. *If wages move symmetrically over the business cycle (same absolute reaction in booms and in recessions:* $\frac{\partial^2 w^I}{\partial a^2} = 0$*), the cutoff will move symmetrically over the business cycle.*

In this case, $\frac{\partial^2 \tilde{\varepsilon}}{\partial a^2} = 0$ and the asymmetry of the selection rate is purely driven by the first term on the right-hand side of equation (28).

Insight 2. *With symmetric wages, the selection rate moves concavely over the business cycle if* $f'\left(\tilde{\varepsilon}\right) < 0$ *(downward sloping part of the density function) and convexly if* $f'\left(\tilde{\varepsilon}\right) > 0$ *(upward sloping part of the density function).*

As $(\frac{\partial \tilde{\varepsilon}}{\partial a})^2 > 0$, the sign of $f'\left(\tilde{\varepsilon}\right)$ determines whether the selection rate is concave or convex. If the cutoff point is in a downward sloping part of the density function (with $f'\left(\tilde{\varepsilon}\right) < 0$), the selection rate moves concavely over the business cycle. In different words, the selection rate reacts less in booms than in recessions. The reason is that with $f'\left(\tilde{\varepsilon}\right) < 0$ a positive aggregate productivity shock shifts the economy to a part of the idiosyncratic training cost distribution with lower density.

So far, we have postulated a symmetric wage for incumbents. With Nash bargaining, the wage for incumbents is given by:

$$w^I = \alpha\left(a + \delta\kappa\theta\right) + \left(1 - \alpha\right) b. \tag{30}$$

In this case, the second derivative of the cutoff point is given by

$$\frac{\partial^2 \tilde{\varepsilon}}{\partial a^2} = \frac{-\frac{\partial^2 w^I}{\partial a^2}}{(1-\delta(1-\phi))(1-\alpha)} = \frac{-\alpha\delta\kappa\frac{\partial^2\theta}{\partial a^2}}{(1-\delta(1-\phi))(1-\alpha)}. \tag{31}$$

Under Nash bargaining, market tightness affects wages. Therefore, we need to look at the second derivative of market tightness:

$$\frac{\partial\theta}{\partial a} = (1-\alpha)\frac{p\eta}{\kappa}\frac{\partial\tilde{\varepsilon}}{\partial a} > 0, \tag{32}$$

$$\frac{\partial^2\theta}{\partial a^2} = \frac{(1-\alpha)\frac{p}{\kappa}f(\tilde{\varepsilon})}{1+\frac{p\alpha\delta}{1-\delta(1-\phi)}}\left(\frac{\partial\tilde{\varepsilon}}{\partial a}\right)^2 > 0. \tag{33}$$

Insight 3. *With Nash bargaining, market tightness reacts in a convex manner ($\frac{\partial^2\theta}{\partial a^2} > 0$) to changes of aggregate productivity, therefore the cutoff point is concave ($\frac{\partial^2\tilde{\varepsilon}}{\partial a^2} < 0$). We will show below that this asymmetry is small for plausible parameter values.*

What does this mean for the selection rate? Remember that

$$\frac{\partial^2\eta}{\partial a^2} = f'(\tilde{\varepsilon})\left(\frac{\partial\tilde{\varepsilon}}{\partial a}\right)^2 + f(\tilde{\varepsilon})\frac{\partial^2\tilde{\varepsilon}}{\partial a^2}. \tag{34}$$

With Nash bargaining the second part on the right-hand side of equation (34) is negative. Being in a downward sloping part of the density function is still a sufficient but no longer necessary condition for a concave selection rate (i.e. $\frac{\partial^2\eta}{\partial a^2} < 0$). If we are in an upward sloping part of the density, we have two countervailing effects.

Insight 4. *Under Nash bargaining, tightness and wages move convexly over the business cycle. Therefore, the region in which the selection rate moves concavely over the business cycle is extended to parts of the density function in which $f'(\tilde{\varepsilon}) > 0$.*

Figure C.3 shows the steady state of the job-finding rate, market tightness, the cutoff point, and the wage for different values of aggregate productivity. We choose the same parameter values as in our baseline calibration but keep the contact rate fixed at its steady state level, i.e. the job-finding rate corresponds to the selection rate multiplied by a constant contact rate. As this is a steady state analysis, we compare permanent differences in productivity and the case with productivity equal to one corresponds to the deterministic steady state in our baseline model. For most of the productivity range, market tightness is almost linear as in the data.[24] Thus, the concavity of the selection rate is almost exclusively driven by the curvature of the underlying density function. Numerically, for a selection rate just below 50% (i.e. the peak of the distribution), the second derivative of the selection rate is practically zero.

[24] Note that earnings per worker are also symmetric over the business cycle in the data.

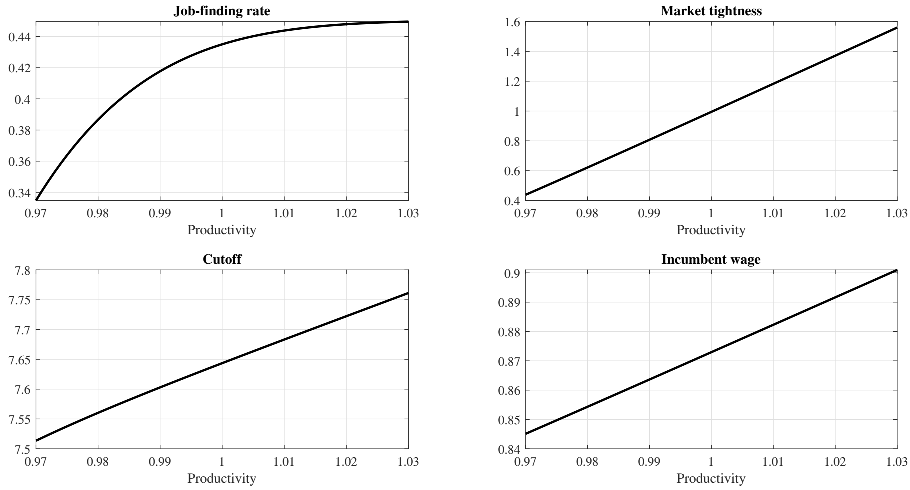

Figure C.3: Steady state values of job-finding rate, market tightness, hiring cutoff and incumbent wage at cutoff for different productivity levels in pure selection model (calibration as in baseline model with fixed contact rate).

C.1.2. The Job-Finding Elasticity

Kohlbrecher et al. (2016) show that there is a positive comovement between the job-finding rate and market tightness in a selection model, which – around the steady state – is observationally equivalent to a Cobb-Douglas constant-returns matching function.

Using the above steady state equations (equations (25), (27), and (32)), they derive the elasticity of the job-finding rate with respect to productivity and the elasticity of market tightness with respect to productivity:

$$\frac{\partial \ln (p\eta)}{\partial \ln a} = \frac{f\left(\tilde{\varepsilon}\right)\frac{\partial \tilde{\varepsilon}}{\partial a}a}{\eta}, \tag{35}$$

$$\frac{\partial \ln \theta}{\partial \ln a} = \frac{\frac{\partial \tilde{\varepsilon}}{\partial a}a}{\tilde{\varepsilon} - \frac{\int_{-\infty}^{\tilde{\varepsilon}} x f(x) dx}{\eta}}. \tag{36}$$

By combining the above equations, the steady state elasticity of the job-finding rate with respect to market tightness (or equivalently of matches with respect to vacancies), i.e. the coefficient usually estimated in a matching function estimation, is given by the following term:

$$\frac{\Delta \ln(p\eta)}{\Delta \ln\theta} = \left(\frac{f(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}a}{\eta}\right) \Big/ \left(\frac{\frac{\partial\tilde{\varepsilon}}{\partial a}a}{\tilde{\varepsilon} - \frac{\int_{-\infty}^{\tilde{\varepsilon}} xf(x)dx}{\eta}}\right)$$

$$= \frac{f(\tilde{\varepsilon})}{\eta}\left(\tilde{\varepsilon} - \frac{\int_{-\infty}^{\tilde{\varepsilon}} xf(x)\,dx}{\eta}\right) \tag{37}$$

$$= \frac{\partial \frac{\int_{-\infty}^{\tilde{\varepsilon}} xf(x)dx}{\eta}}{\partial\tilde{\varepsilon}}. \tag{38}$$

This term only depends on the distribution of the idiosyncratic shock and the position of the cutoff point. Therefore, the shape of the distribution around the cutoff point determines the elasticity of the job-finding rate with respect to market tightness in this model. Intuitively, the elasticity corresponds to the first derivative of the conditional expectation of idiosyncratic training costs with respect to the cutoff point.

Two remarks are in order: First, the formula is quite general: It holds for a very general wage rule (see equation (21), note that $\alpha = 0$ is admissible). In addition, Kohlbrecher et al. (2016) show that the formula is the same for permanent match productivity and endogenous separations.[25] Second, the link between the job-finding rate and market tightness is not causal in this model. The derived elasticity rather reflects a joint comovement of the two variables in response to productivity changes.

A graphical illustration for the comovement between the job-finding rate and market tightness is shown in Figure 9. The elasticity of the job-finding rate with respect to market tightness, i.e. the implied weight on vacancies in an estimated matching function, changes with the position of the cutoff point. If the cutoff point is located at a downward sloping part of the density function, a negative aggregate productivity shock moves it to a part of the distribution with more density, which causes the asymmetric response of the job-finding rate. At the same time, the decline of productivity changes the relationship between the job-finding rate and market tightness (see lower panel in in Figure 9). The elasticity of the job-finding rate with respect to market tightness increases.

This observation gives us two important insights.

Insight 5. *The selection model implies a decline in matching efficiency in recessions if seen through the lens of a constant-elasticity matching function.*

In recessions, the model-implied elasticity between the job-finding rate and market tightness increases. This is not accounted for if we estimate a time-invariant matching function. Therefore, it appears as if the job-finding rate has fallen too much for a given change in market tightness, i.e. the estimated matching efficiency – defined as the residual from a constant-elasticity matching function – decreases when aggregate productivity drops.

[25] If the idiosyncratic training cost shock was multiplicative instead of additive, the elasticity would be given not by the simple derivative of the conditional expectation with respect to the cutoff point but by the corresponding log-log derivative.

Insight 6. *For empirically plausible values for the elasticity of matches with respect to vacancies, the hiring cutoff needs to be in the downward-sloping part of the distribution.*

For all cases in Figure 9, the elasticity curves cross the 0.5-line after the peak of the distribution. A weight on vacancies in a matching function of 0.5 is often seen as an upper bound in the empirical literature (see Petrongolo and Pissarides, 2001). Therefore, in order to be consistent with the empirical matching function estimates, we require a cutoff point in the downward sloping part of the distribution. Note that this condition is based on a pure selection model in which the job-finding rate is only driven by the selection rate, i.e. the elasticity of the contact rate with respect to market tightness is zero. Adding a pro-cyclical contact rate makes our argument even stronger, as this adds to the overall elasticity of the job-finding rate with respect to market tightness.

C.2. Search and Matching Model

We now show under which conditions a search and matching model without selection can generate a concave job-finding rate. The vacancy-free entry condition of the standard search and matching model equates the vacancy posting costs per hire to the discounted present value of match profits:

$$\frac{\kappa}{q(\theta)} = a - w + \delta(1-\phi)\frac{\kappa}{q(\theta)}. \tag{39}$$

By iterating forward, we get the following steady state expression for profits:

$$\pi = \frac{a-w}{1-\delta(1-\phi)}. \tag{40}$$

Substituting back into (39), we can express the worker-finding rate as a function of the vacancy posting costs and discounted profits only:

$$q(\theta) = \frac{\kappa}{\pi}. \tag{41}$$

With a Cobb-Douglas matching function, the contact rate, which now equals the job-finding rate, is defined as follows (for ease of exposition we set the efficiency parameter μ to unity):

$$p(\theta) = \theta^{\gamma}. \tag{42}$$

For any $\gamma \in (0,1)$ the contact rate is a concave function of tightness, where γ is the elasticity of matches with respect to vacancies. However, in order to determine whether the contact rate actually exhibits asymmetry in the model, one has to account for the equilibrium movement of tightness. We therefore combine equation (41) with the definition of the worker-finding rate to derive market tightness in equilibrium:

$$\theta = q(\theta)^{\frac{1}{\gamma-1}} = \left(\frac{\kappa}{\pi}\right)^{\frac{1}{\gamma-1}} = \left(\frac{\pi}{\kappa}\right)^{\frac{1}{1-\gamma}}. \tag{43}$$

C.2.1. Business Cycle Asymmetries

The first and second derivative of the contact rate with respect to productivity are:

$$\frac{\partial p}{\partial a} = \gamma \theta^{\gamma-1} \frac{\partial \theta}{\partial a} \tag{44}$$

and

$$\frac{\partial^2 p}{\partial a^2} = (\gamma - 1)\gamma \theta^{\gamma-2} \left(\frac{\partial \theta}{\partial a}\right)^2 + \gamma \theta^{\gamma-1} \frac{\partial^2 \theta}{\partial a^2}. \tag{45}$$

The first term on the right-hand side of equation (45) is negative. It reflects the fact that the contact rate is a concave function of tightness. The sign of the second term depends on whether tightness is concave or convex in productivity.

The first derivative of tightness with respect to productivity is given by:

$$\begin{aligned} \frac{\partial \theta}{\partial a} &= \frac{1}{1-\gamma} \left(\frac{a-w}{\kappa(1-\delta(1-\phi))}\right)^{\frac{\gamma}{1-\gamma}} \frac{1-\frac{\partial w}{\partial a}}{\kappa(1-\delta(1-\phi))} \\ &= \frac{1}{1-\gamma} \theta^{\gamma} \frac{1-\frac{\partial w}{\partial a}}{\kappa(1-\delta(1-\phi))}, \end{aligned} \tag{46}$$

where we have made use of equation (43) in the second step. The second derivative is given by

$$\begin{aligned} \frac{\partial^2 \theta}{\partial a^2} =& \frac{1}{1-\gamma} \frac{\gamma}{1-\gamma} \left(\frac{a-w}{\kappa(1-\delta(1-\phi))}\right)^{\frac{2\gamma-1}{1-\gamma}} \left(\frac{1-\frac{\partial w}{\partial a}}{\kappa(1-\delta(1-\phi))}\right)^2 \\ &+ \frac{1}{1-\gamma} \left(\frac{a-w}{\kappa(1-\delta(1-\phi))}\right)^{\frac{\gamma}{1-\gamma}} \frac{-\frac{\partial^2 w}{\partial a^2}}{\kappa(1-\delta(1-\phi))}, \end{aligned} \tag{47}$$

which again simplifies to:

$$\frac{\partial^2 \theta}{\partial a^2} = \frac{\gamma}{(1-\gamma)^2} \theta^{2\gamma-1} \left(\frac{1-\frac{\partial w}{\partial a}}{\kappa(1-\delta(1-\phi))}\right)^2 + \frac{1}{1-\gamma} \theta^{\gamma} \frac{-\frac{\partial^2 w}{\partial a^2}}{\kappa(1-\delta(1-\phi))}. \tag{48}$$

Insight 1. *If wages are symmetric, i.e.* $\frac{\partial^2 w}{\partial a^2} = 0$*, market tightness is a convex function of productivity.*

With symmetric wages, equation (48) simplifies to

$$\frac{\partial^2 \theta}{\partial a^2} = \frac{\gamma}{(1-\gamma)^2} \theta^{2\gamma-1} \left(\frac{1-\frac{\partial w}{\partial a}}{\kappa(1-\delta(1-\phi))}\right)^2 > 0. \tag{49}$$

Insight 2. *If wages are symmetric, i.e.* $\frac{\partial^2 w}{\partial a^2} = 0$*, the contact rate is symmetric for* $\gamma = 0.5$*, concave for* $\gamma < 0.5$ *and convex for* $\gamma > 0.5$*.*

Plugging equations (46) and (49) into equation (45) we get:

$$\frac{\partial^2 p}{\partial a^2} = \frac{\gamma}{(1-\gamma)^2}\theta^{3\gamma-2}\left(\frac{1-\frac{\partial w}{\partial a}}{\kappa(1-\delta(1-\phi))}\right)^2(2\gamma-1) \tag{50}$$

Thus, under symmetric wages the second derivative of the contact rate with respect to productivity is exactly zero if unemployment and vacancies have equal weight in the matching function, i.e. $\gamma = 0.5$. Intuitively, the convexity of tightness completely offsets the concave shape of the contact rate with respect to tightness. The first and second term on the right-hand side of equation (45) cancel out. However, if the weight on vacancies is smaller ($\gamma < 0.5$), the second derivative is negative and the contact rate will react stronger in recessions than in booms. If $\gamma > 0.5$, the opposite is the case and the contact rate will show a stronger reaction to a positive productivity shock.

So far we have assumed symmetric wages. Under Nash bargaining, however, the wage is $w = \alpha(a + \delta\kappa\theta) + (1-\alpha)(1-b)$. The second derivative of wages with respect to productivity is $\alpha\delta\kappa\frac{\partial^2\theta}{\partial a^2}$. Plugging this into equation (48) and rearranging gives:

$$\frac{\partial^2 \theta}{\partial a^2} = \frac{\frac{\gamma}{(1-\gamma)^2}\theta^{2\gamma-1}\left(\frac{1-\frac{\partial w}{\partial a}}{\kappa(1-\delta(1-\phi))}\right)^2}{1+\frac{\gamma}{1-\gamma}\frac{\alpha\delta\kappa}{\kappa(1-\delta(1-\phi))}} > 0. \tag{51}$$

Insight 3. *Under Nash bargaining, market tightness is convex in productivity but to a lesser degree than under symmetric wages.*

The denominator of equation (51) is strictly larger than 1. This implies that the second derivative under Nash bargaining (equation (51)) is positive but strictly smaller than the corresponding value under symmetric wages (equation (49)).

Insight 4. *Under Nash bargaining the contact rate is concave even for $\gamma = 0.5$. The degree of concavity is decreasing in γ.*

This follows directly from the preceding analysis. Remember that for symmetric wages and $\gamma = 0.5$ the convexity of tightness just canceled with the concave shape of the contact function. Given that the convexity of tightness is dampened under Nash bargaining, the second derivative of the contact rate with respect to productivity will be negative for $\gamma = 0.5$ and some region above. Intuitively, as wages are a function of tightness, wages are convex in productivity as well, which negatively feeds back into the second derivative of tightness with respect to productivity. As a general rule, the lower γ, i.e. the lower the elasticity of contacts (matches) with respect to vacancies, the higher the curvature of the job-finding rate.

Figure C.4 shows the steady state of the job-finding rate, market tightness, vacancies and wages for different values of aggregate productivity in a standard search and matching model. The parameters are chosen to generate realistic amplification of the job-finding rate and the elasticity of the contact (i.e. job-finding) rate is set to 0.28 as in the data (the calibration is described in Section E). As expected, the job-finding rate is clearly concave.

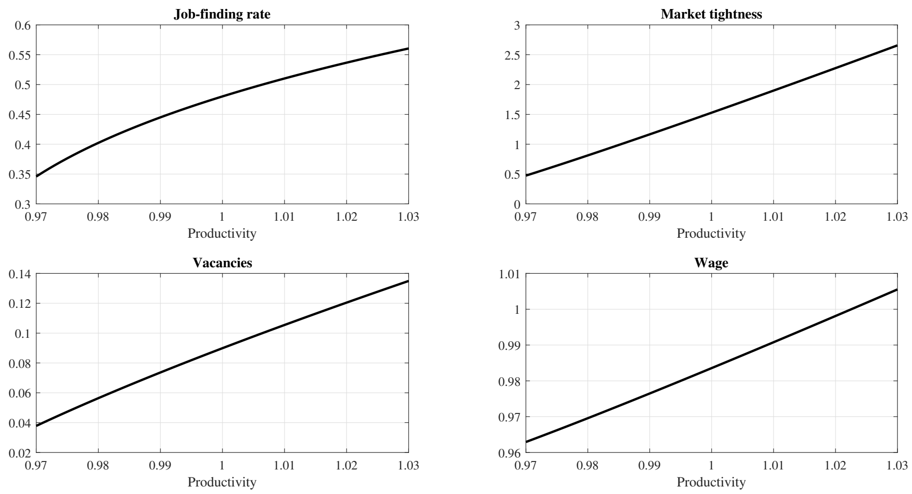

Figure C.4: Steady state values of job-finding (= contact) rate, market tightness, vacancies and wages for different productivity levels in standard search and matching model (calibration as described in Section E).

C.2.2. The Job-Finding Elasticity

In a standard search and matching model without labor selection, the job-finding rate is determined by a constant elasticity matching function. By definition, matching efficiency cannot shift endogenously.

D. Solution of the model

We solve the model fully stochastically and fully nonlinearly. For this purpose it is convenient to cast the model in a slightly different notation.[26]

D.1. Model Description

Let $M_t^E(\varepsilon_t)$ denote the match value and $S_t^E(\varepsilon_t)$ the match surplus of a newly formed match with idiosyncratic training costs ε_t. M_t and S_t denote the match value and surplus of incumbent worker-firm matches. Whenever a new contact has a positive match surplus, i.e. $S_t^E(\varepsilon_t) > 0$, the match is created. Because the surplus is strictly decreasing in the training costs realization, there will be a unique selection cutoff $\tilde{\varepsilon}_t$, such that $S_t^E(\tilde{\varepsilon}_t) = 0$ and only workers below that threshold will be selected. Per construction, incumbent matches always have a positive match surplus in our baseline model, i.e. there are no endogenous separations. With Nash bargaining, workers and firms receive a constant share of the match surplus as payoff. The bargaining power of workers is α. Then, the

[26] We have opted for a different representation of the model in the main part of the paper as it highlights the role of the selection rate.

match payoff for a firm is $\pi_t^I = (1-\alpha)S_t + V_t$ and $\pi_t^E(\varepsilon_t) = (1-\alpha)S_t^E(\varepsilon_t) + V_t$. The worker's payoff is $W_t^I = \alpha S_t + U_t$ and $W_t^E(\varepsilon_t) = \alpha S_t^E(\varepsilon_t) + U_t$.

The value of unemployment can be expressed as follows:

$$U_t = b + \delta \mathbb{E}_t \left[p(\theta_{t+1})\alpha \int_{-\infty}^{\infty} S_{t+1}^E(x) f(x) dx + U_{t+1} \right]. \tag{52}$$

Note that as we now integrate over the unconditional surpluses, the selection rate no longer appears as a variable.

The value of a vacancy is

$$V_t = -\kappa + q(\theta_t)(1-\alpha) \int_{-\infty}^{\infty} S_t^E(x) f(x) dx. \tag{53}$$

With the value of a vacancy driven to zero by free-entry, we get:

$$\frac{\kappa}{q(\theta_t)} = (1-\alpha) \int_{-\infty}^{\infty} S_t^E(x) f(x) dx. \tag{54}$$

The joint value of a match for a new worker-firm contact is

$$M_t^E(\varepsilon_t) = \max \left\{ M_t^{E,m}(\varepsilon_t), U_t + V_t \right\} \tag{55}$$

where the subscript m represents the value when the match is formed, i.e. the worker is selected:

$$M_t^{E,m}(\varepsilon_t) = a_t - \varepsilon_t + \delta \mathbb{E}_t \left[(1-\phi) M_{t+1} + \phi (U_{t+1} + V_{t+1}) \right]. \tag{56}$$

The value of an incumbent match is simply

$$M_t = a_t + \delta \mathbb{E}_t \left[(1-\phi) M_{t+1} + \phi (U_{t+1} + V_{t+1}) \right]. \tag{57}$$

By subtracting U_t and V_t from (55) and (57) and using the fact that (due to the free-entry condition) $V_t = 0$, we obtain:

$$S_t^E(\varepsilon_t) = \max \left\{ S_t^{E,m}(\varepsilon_t), 0 \right\}, \tag{58}$$

$$S_t^{E,m}(\varepsilon_t) = a_t - \varepsilon_t - b + \delta \mathbb{E}_t \left[(1-\phi) S_{t+1} - p(\theta_{t+1})\alpha \int_{-\infty}^{\infty} S_{t+1}^E(x) f(x) dx \right], \tag{59}$$

and

$$S_t = a_t - b + \delta \mathbb{E}_t \left[(1-\phi) S_{t+1} - p(\theta_{t+1})\alpha \int_{-\infty}^{\infty} S_{t+1}^E(x) f(x) dx \right]. \tag{60}$$

D.2. Solution

For given realizations of a_t, the equilibrium is given by (54), (58), and (60). This system can be solved by backward substitution as described e.g. in Fujita and Ramey (2012). We estimate an AR(1) process from our monthly productivity growth data (see Section 5) and recast this process as a Markov chain with N=1001 grid points using the method by Tauchen (1986).[27] The distribution for idiosyncratic training costs is represented by a truncated normal distribution with K=1000 equally spaced grid points with support $\{\mu - 6\sigma_\varepsilon, ..., \mu + 10\sigma_\varepsilon\}$.[28] We can now solve for $N \times 1$ values of tightness and the associated $N \times K$ values of the match surpluses by backward substitution. The selection cutoffs at every state can then be retrieved by finding the grid point of training costs at which the surplus turns zero.

For the dynamic simulation, we let the aggregate states of the model track the evolution of the actual productivity growth series in the data, i.e. the productivity series fed into our model corresponds one to one to annualized productivity growth in the data. For the nonlinear impulse responses the discrete states follow the evolution of an initial 2.5% shock to the AR(1) process for productivity. In each of our exercises, we let the model evolve for 200 quarters with aggregate productivity set to its steady state value such that the model rests at its stochastic steady state at the start of any exercise.[29]

E. Comparison with Standard Search and Matching Model

A classical search and matching model without a selection margin can also display asymmetry in the job-finding rate and unemployment if it is calibrated to generate enough amplification (Petrosky-Nadeau and Zhang, 2017; Petrosky-Nadeau et al., 2018). How far does a standard search and matching model get in terms of asymmetry compared to our model with selection? For this purpose we calibrate our model without idiosyncratic training costs, which brings us back to a textbook search and matching model. For comparability, we again target a mean job-finding rate of 0.42 and a mean market tightness of 1. The elasticity of the job-finding rate – which is now equal to the contact rate – is set to 0.28. The exogenous separation rate is 0.03. For a fair comparison, we calibrate the search and matching model to generate the same amount of amplification as our baseline model. Specifically, we match the average absolute impact response of the job-finding rate in response to a 2.5% negative and positive productivity shock in the selection model. We achieve this by setting the unemployment compensation to $b = 0.954$ and the bargaining power of workers to $\alpha = 0.05$ in the search and matching

[27] A high number of grid points is convenient as it allows for smooth impulse responses and a close mapping between the productivity series in the data and the model. An alternative would be to choose a low number of grid points and then to interpolate the solution. As computation is fast, we opted for the first option.

[28] We choose a relatively higher maximum support, as the training costs cutoff is situated above the mean. Having a high maximum support ensures good precision even for large responses of the cutoff.

[29] Note that because of the high amplification and nonlinearity of the model, the deterministic steady state, the stochastic steady state and the ergodic mean of the labor market variables can differ substantially.

model. These values are very close to the original parameter values in Hagedorn and Manovskii (2008), i.e. we apply a small-surplus calibration in order to get amplification of the job-finding rate.

Figure E.5 shows the impulse responses of the job-finding and unemployment rate of our selection model (solid lines) and the standard search and matching model (dashed lines) in response to a 2.5% negative and positive productivity shock. Note that we have flipped the negative responses (in red) for better visibility. Figure E.5 clearly shows that the degree of asymmetry generated by the search and matching model (conditional on the same amount of amplification of the job-finding rate) is a lot smaller than in our selection model. In addition, Figure E.6 shows that the shift of the Beveridge curve in booms and recessions is also far more symmetric in the search and matching model (dashed line). In the search and matching model matching efficiency is constant and cannot shift except for exogenous reasons. This leads to a more symmetric trajectory of the Beveridge curve.

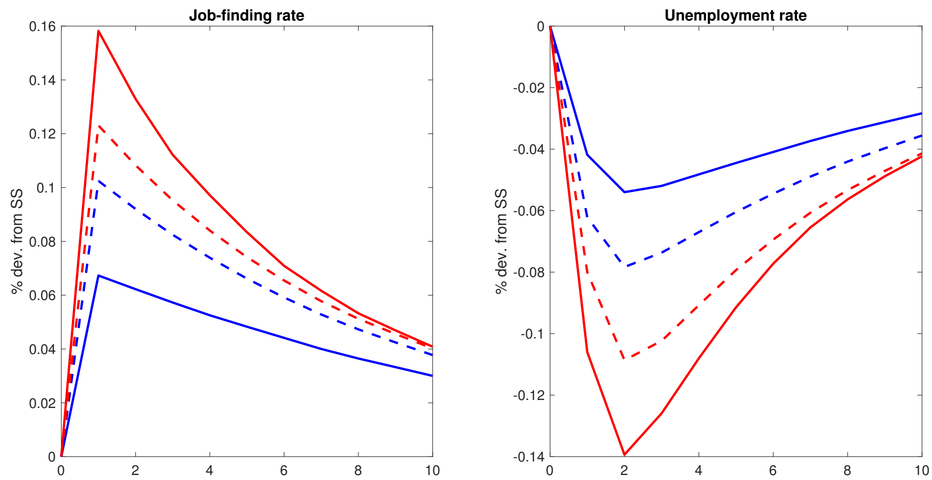

Figure E.5: Model impulse responses to 2.5% positive and negative productivity shock (in percent deviations from steady state) for baseline selection model (solid lines) and standard search and matching model (dashed lines). Responses to negative shock (in red) are flipped for better visibility.

F. Model Extension: Persistent Shocks and Endogenous Separations

In the main part of the paper we have kept the model simple in order to highlight the core mechanism – selection at the hiring stage – and its role for job-creation. However, it is possible that in reality idiosyncratic training costs – or more generally idiosyncratic productivity – are more persistent in a match and that idiosyncratic costs also play a

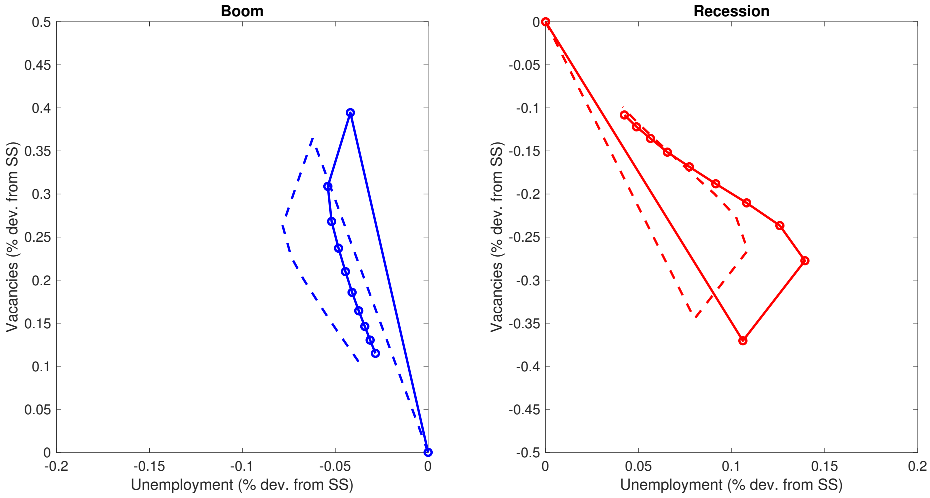

Figure E.6: Beveridge curve shift in boom and recession scenario for baseline selection model (solid line) and standard search and matching model (dashed line).

role for job destruction. We now check whether our baseline results are robust to the following model extensions: We make idiosyncratic training costs persistent and also subject existing worker-firm matches to idiosyncratic cost shocks. Both features can lead to endogenous separations, either because the existing idiosyncratic cost realization exceeds the new period's firing cutoff or because a new draw makes matches unprofitable. The model is closely related to the model with endogenous separations in Fujita and Ramey (2012) but enriches it with a selection decision for new hires.[30]

F.1. Model Description

As before, all new contacts draw a realization, ε, from an idiosyncratic cost distribution with density $f(\varepsilon)$.[31] This match-specific cost realization is permanent until the match draws a new realization which happens with probability λ. A match can be destroyed for two reasons: Whenever the idiosyncratic cost realization lies above the current period's firing threshold, $\tilde{\varepsilon}^I_t$, the match is destroyed endogenously. We call the associated (unconditional) firing probability ϕ_t. In addition, in every period some matches end exogenously with probability s. We assume that both new and existing matches draw from the same invariant cost distribution and every draw is *iid*. New matches also pay a fixed training cost, tc, in the first period of production. This assumption drives a wedge between the selection rate for new hires and the endogenous separation rate for existing matches. This is necessary as the former is calibrated to the elasticity of the job-finding rate with respect to market tightness while the latter is calibrated to match the mean

[30]In Fujita and Ramey (2012) new matches always start at the highest value of match productivity and hence all new contacts are transformed into matches.

[31]We could equally define the shock positively, i.e. as a match specific productivity.

separation rate in the data.

The expected discounted profit of a firm hiring a new worker with idiosyncratic cost ε_t is given by:

$$\begin{aligned}\pi_t^E(\varepsilon_t) = a_t - tc - \varepsilon_t - w_t^E(\varepsilon_t) + \delta\mathbb{E}_t \big[\lambda(1-s)(1-\phi_{t+1})\pi_{t+1}^I(\varepsilon_{t+1}|\varepsilon_{t+1} \leq \tilde{\varepsilon}_{t+1}^I) \\ +(1-\lambda)(1-s)I_{t+1}^f \pi_{t+1}^I(\varepsilon_t)\big].\end{aligned} \tag{61}$$

The expected discounted profit of a firm with an incumbent worker with idiosyncratic cost ε_t is:

$$\begin{aligned}\pi_t^I(\varepsilon_t) = a_t - \varepsilon_t - w_t^I(\varepsilon_t) + \delta\mathbb{E}_t \big[\lambda(1-s)(1-\phi_{t+1})\pi_{t+1}^I(\varepsilon_{t+1}|\varepsilon_{t+1} \leq \tilde{\varepsilon}_{t+1}^I) \\ +(1-\lambda)(1-s)I_{t+1}^f \pi_{t+1}^I(\varepsilon_t)\big].\end{aligned} \tag{62}$$

The superscripts E and I refer to values associated with entrants (i.e. new hires) and incumbent workers. I_{t+1}^f is an indicator function that takes the value of one whenever a match that keeps its idiosyncratic cost realization from last period continues production and zero otherwise:

$$I_{t+1}^f = \begin{cases} 1 & \text{if} \quad \varepsilon_t \leq \tilde{\varepsilon}_{t+1}^I \\ 0 & \text{otherwise} \end{cases}. \tag{63}$$

The associated cutoff values that ensure that $\pi_t^E(\varepsilon_t) = 0$ and $\pi_t^I(\varepsilon_t) = 0$ are

$$\begin{aligned}\tilde{\varepsilon}_t^E = a_t - tc - w_t^E(\tilde{\varepsilon}_t^E) + \delta\mathbb{E}_t \big[\lambda(1-s)(1-\phi_{t+1})\pi_{t+1}^I(\varepsilon_{t+1}|\varepsilon_{t+1} \leq \tilde{\varepsilon}_{t+1}^I) \\ +(1-\lambda)(1-s)I_{t+1}^f \pi_{t+1}^I(\tilde{\varepsilon}_t^E)\big],\end{aligned} \tag{64}$$

and

$$\begin{aligned}\tilde{\varepsilon}_t^I = a_t - w_t^I(\tilde{\varepsilon}_t^I) + \delta\mathbb{E}_t \big[\lambda(1-s)(1-\phi_{t+1})\pi_{t+1}^I(\varepsilon_{t+1}|\varepsilon_{t+1} \leq \tilde{\varepsilon}_{t+1}^I) \\ +(1-\lambda)(1-s)I_{t+1}^f \pi_{t+1}^I(\tilde{\varepsilon}_t^I)\big].\end{aligned} \tag{65}$$

The selection rate is thus

$$\eta_t = \int_{-\infty}^{\tilde{\varepsilon}_t^E} f(x)d(x) \tag{66}$$

and the endogenous firing rate is

$$\phi_t = \int_{\tilde{\varepsilon}_t^I}^{\infty} f(x)d(x). \tag{67}$$

Finally, the free-entry condition for vacancies is:

$$\frac{\kappa}{q_t \eta_t} = \mathbb{E}_t \left[\pi_t^E(\varepsilon_t|\varepsilon_t \leq \tilde{\varepsilon}_t^E)\right]. \tag{68}$$

F.2. Calibration

The solution of the model follows the method described in Appendix D. We apply the same calibration strategy as in the model with exogenous separations. We target a mean job-finding rate of 0.42, a separation rate of 0.03 (now defined as total employment to unemployment flows over employment) and a market tightness normalized to 1. In addition, we match the elasticity of the job-finding rate with respect to market tightness of 0.28 and set the elasticity of the contact rate with respect to vacancies/tightness to 0.1. We assume that exogenous separations account for 2/3 of separations (see e.g. Krause and Lubik, 2007) and set the exogenous separation rate to 0.02. Fujita and Ramey (2012) set the probability of drawing a new shock realization to 0.085 in their weekly calibration to replicate the empirical tenure distribution. We follow them and set $\lambda = 0.3$ in our monthly calibration. We choose the same unemployment benefits as in the baseline model ($b = 0.7$) and the same standard deviation for the idiosyncratic training cost distribution (0.11). We are left with one free parameter, namely the bargaining power of workers, which we use to match the volatility of the job-finding rate relative to productivity, which is 5.6. We achieve this by setting $\alpha = 0.05$.

F.3. Results

Figure F.7 shows the impulse responses to a 2.5% positive and negative productivity shock for the model with persistent cost shocks and endogenous separations. What stands out is that the responses of the job-finding rate, the selection rate, the contact rate, and tightness are all quantitatively and qualitatively very similar to the IRFs in the baseline model with exogenous separations and one-period shocks. The model thus generates an asymmetric job-finding rate, which is predominantly driven by an asymmetric selection rate. One of the key facts from our analysis is therefore confirmed. The most striking difference is the response of vacancies to a negative shock. Instead of declining, they stay relatively constant on impact and then even increase. This is the result of endogenous separations. Indeed, separations are overly volatile in our calibration compared to the data,[32] and the excessive spike in separations in response to a negative shock makes the Beveridge curve collapse. This is a well known feature of models with endogenous separations. As a result of the excessive spike in separations, the asymmetric response of the unemployment rate is reinforced.

Although the Beveridge curve collapses, our second key fact – the decline of the matching efficiency in recessions – still stands. This can be seen in Figure F.8. When tightness is very low, the job-finding rate is systematically below the predicted value from a Cobb-Douglas matching function estimation. The Cobb-Douglas function is not able to capture all the curvature produced by the model, which – through the lens of this function – would appear as a drop in matching efficiency in severe recessions.

To conclude, our main results – that a model with selection upon hiring generates an asymmetric job-finding rate and a drop of matching efficiency in severe recessions

[32] The standard deviation of the separation rate is more than 30 times larger than the standard deviation of aggregate productivity.

– is robust to endogenous separations and persistent idiosyncratic shocks. In addition, endogenous separations add to the asymmetry of the unemployment rate. One disadvantage of this model version is the disappearance of the Beveridge curve, which is clearly at odds with the data. This is because separations are excessively volatile compared to the data. This could be addressed by including on-the-job-search and thus endogenous quits. Given that these are not decisive for our key results and would greatly increase complexity, we refrain from such an exercise.

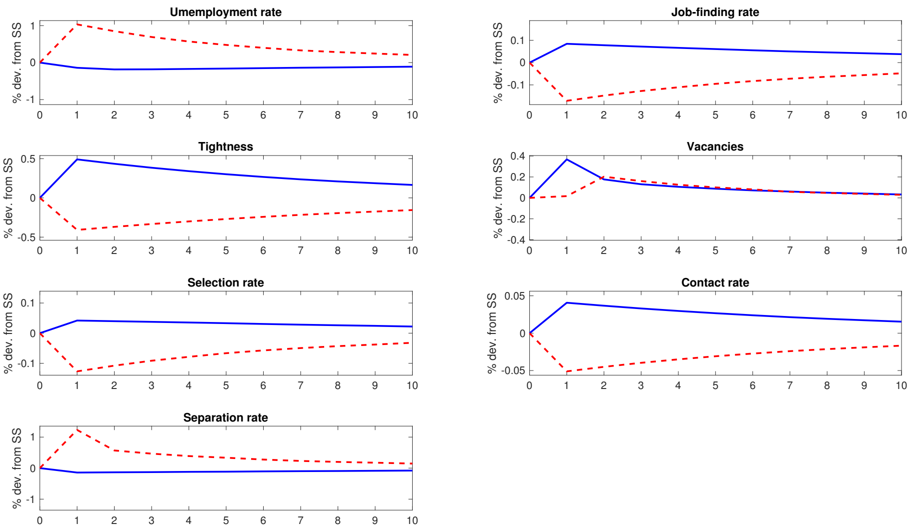

Figure F.7: Impulse responses to a 2.5% positive (solid blue line) and a 2.5% negative (dashed red line) productivity shock (in percent deviations from steady state) in model with persistent shocks and endogenous separations.

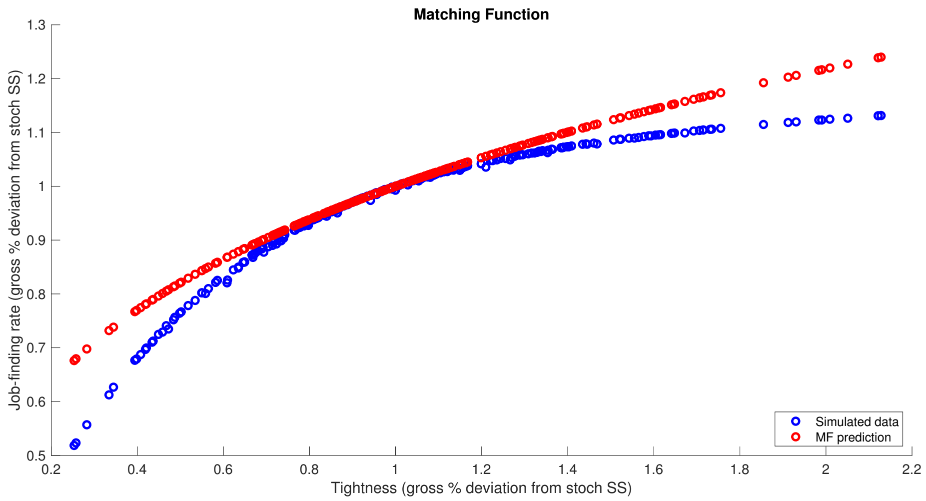

Figure F.8: The model-based matching function.